COMPLETING THE SQUARE

The following 4 examples use the method of completing the square to rewrite an expression. Nothing is being "solved" here so we add and subtract a special quantity (the net change is 0 and we haven't changed the expression). The special quantity is $\left(\frac{coefficient\ of\ x}{2}\right)^2$. If the coefficient of x^2 is not 1, you should factor that coefficient out of both the x^2 and x terms. A common mistake is to divide though by that coefficient, but this is not an equation so you can't do that without changing the problem.

$$\begin{aligned} x^2 - 6x + 7 &= x^2 - 6x + 9 - 9 + 7 \\ &= \left(x^2 - 6x + 9\right) - 9 + 7 \\ &= (x-3)^2 - 2 \end{aligned}$$

$$\begin{aligned} x^2 + bx + c &= x^2 + bx + \frac{b^2}{4} - \frac{b^2}{4} + c \\ &= \left(x^2 + bx + \frac{b^2}{4}\right) - \frac{b^2}{4} + c \\ &= \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c \end{aligned}$$

$$\begin{aligned} 2x^2 + 10x + 3 &= 2\left(x^2 + 5x\right) + 3 \\ &= 2\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4}\right) + 3 \\ &= 2\left(x^2 + 5x + \frac{25}{4}\right) - 2\cdot\frac{25}{4} + 3 \\ &= 2\left(x + \frac{5}{2}\right)^2 - \frac{19}{2} \end{aligned}$$

$$\begin{aligned} ax^2 + bx + c &= a\left(x^2 + \frac{b}{a}x\right) + c \\ &= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c \\ &= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - a\cdot\frac{b^2}{4a^2} + c \\ &= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c \end{aligned}$$