

CRITICAL POINTS (4.1)

NAME _____

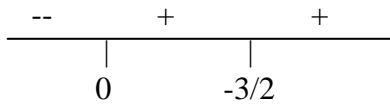
4. Use Calculus to determine i) critical points, ii) local extrema, iii) inflection points, and iv) intervals where $f(x)$ is concave up or down.

A. $f(x) = x^4 + 2x^3 - 1$

$$f'(x) = 4x^3 + 6x^2$$

$$0 = 2x^2(2x + 3)$$

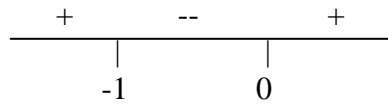
$$x = 0, x = -\frac{3}{2}$$



$$f''(x) = 12x^2 + 12x$$

$$0 = 12x(x + 1)$$

$$x = 0, x = -1$$

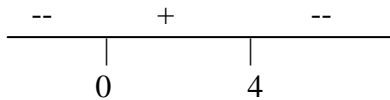


B. $f(x) = \frac{8x-16}{x^2} = 8x^{-1} - 16x^{-2}$

$$f'(x) = -8x^{-2} + 32x^{-3} = \frac{-8x + 32}{x^3}$$

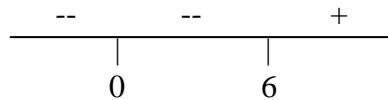
$x = 4$ is a stationary point

$x = 0$ is a vertical asymptote



$$f''(x) = 16x^{-3} - 96x^{-4} = \frac{16x - 96}{x^4}$$

$$x = 6, x = 0$$

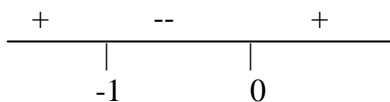


C. $f(x) = 2x + 3x^{2/3}$

$$f'(x) = 2 + 2x^{-1/3} = \frac{2x^{1/3} + 2}{x^{1/3}}$$

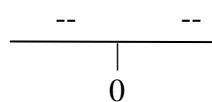
$x = -1$ is a stationary point

$x = 0$ is a point of ND



$$f''(x) = -\frac{2}{3}x^{-4/3}$$

$$x = 0$$



MORE PROBLEMS

1. $f(x) = 6x^{2/3} + 3x^{5/3}$

$$f'(x) = 4x^{-1/3} + 5x^{2/3} = \frac{4+5x}{x^{1/3}}$$

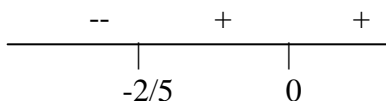
$x = -\frac{4}{5}$ is a stationary point

$x = 0$ is a point of ND



$$f''(x) = -\frac{4}{3}x^{-4/3} + \frac{10}{3}x^{-1/3} = \frac{4+10x}{3x^{4/3}}$$

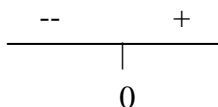
$x = -\frac{2}{5}, x = 0$



2. $h(x) = 6x^{2/3} + 3x^{8/3}$

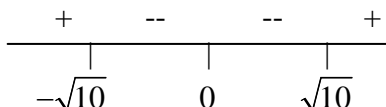
$$h'(x) = 4x^{-1/3} + 8x^{5/3} = \frac{4+8x^2}{x^{1/3}}$$

$x = 0$ is a point of ND



$$h''(x) = -\frac{4}{3}x^{-4/3} + \frac{40}{3}x^{2/3} = \frac{-4+40x^2}{3x^{4/3}}$$

$x = \pm\sqrt{10}, x = 0$



3. $g(x) = x^2 \ln(x)$

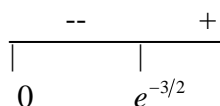
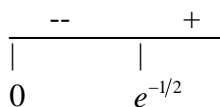
Note: The domain is $x > 0$ and $\lim_{x \rightarrow 0^+} (x^2 \ln(x)) = 0$ means there is a hole at $(0,0)$

$$g'(x) = x^2 \cdot \frac{1}{x} + \ln(x) \cdot 2x = x + 2x \ln(x) = x(1 + 2 \ln(x))$$

$x = e^{-1/2}$ is a stationary point

$$g''(x) = 1 + 2x \cdot \frac{1}{x} + \ln(x) \cdot 2 = 3 + 2 \ln(x)$$

$x = e^{-3/2}$



$$4. p(x) = e^{1/x}$$

Note: $\lim_{x \rightarrow 0^-} e^{1/x} = 0$ and $\lim_{x \rightarrow 0^+} e^{1/x} = \infty$

$$p'(x) = e^{1/x} \cdot \frac{-1}{x^2}$$

$$p''(x) = e^{1/x} \cdot 2x^{-3} + x^{-2} e^{1/x} x^{-2} \cdot = e^{1/x} \left(\frac{2}{x^3} + \frac{1}{x^4} \right) = e^{1/x} \left(\frac{2x+1}{x^4} \right)$$

