TAYLOR SERIES (practice)

1. The following parts refer to $f(x)=\dfrac{1}{3-x}$.

A. Write the Taylor series expansion for $f(x)$ about $x = 0$.

B. Expand $f(x)$ in a Taylor series in terms of $x/3$ about $x = 0$.

C. Find the Taylor series expansion for $f(x)$ about $x = 2$ without actually taking the derivatives. (Hint: rewrite the denominator so that $x-2$ appears.)

D. Do part C by actually taking the derivatives of $f(x)$. Is your series expansion identical to the one you got in part C?

E. Use part A to find each of the following $f'(0)$ $f'''(0)$ $f^{(5)}(0)$

F. Use part C to find each of the following $f'(2)$ $f'''(2)$ $f^{(5)}(2)$

G. Find the Taylor series expansion about $x=0$ for $\dfrac{x}{3-x}$. Include at least four nonzero terms.

2. Find the Taylor series about 0 for the functions below. Include at least four nonzero terms.

A. $f(x)=\arcsin x$ B. $f(x)=\ln(3x+1)$ C. $f(x)=\dfrac{\sin(x^2)}{x}$

3. A. Find the Taylor polynomial of degree 3 approximating $f(x)=\sqrt{1-x}$ near 0.

B. Use the polynomial in part A to give approximate values of $\sqrt{0.5}$ and $\sqrt{0.9}$.

C. Which approximation in part B is more accurate? Why?

4. A. Suppose all the derivatives of a function f exist at 0. If the Taylor series for f about $x=0$ is given by $f(x)=5x^3-7x^5+9x^7-\cdots$ find $f^{(4)}(0)$ $f^{(5)}(0)$ $f^{(7)}(0)$

B. If $f(x)=\sum_{n=1}^{\infty}\dfrac{(-1)^n(x-\pi)^n}{n^2}$ find $f'''(\pi)$

5. Find the exact value of the following sums:

A. $5-\frac{5(0.2)^2}{2!}+\frac{5(0.2)^4}{4!}-\frac{5(0.2)^6}{6!}+\frac{5(0.2)^8}{8!}-\cdots$

B. $(0.2)^2-\frac{(0.2)4}{3!}+\frac{(0.2)^6}{5!}-\frac{(0.2)^8}{7!}+\cdots$

C. $5-\frac{5(0.2)^2}{1!}+\frac{5(0.2)^4}{2!}-\frac{5(0.2)^6}{3!}+\frac{5(0.2)^8}{4!}-\cdots$

D. $\frac{5(0.2)^2}{2}+\frac{5(0.2)^4}{2}+\frac{5(0.2)^6}{2}+\frac{5(0.2)^8}{2}-\cdots$

E. $5+5+\frac{5}{2!}+\frac{5}{3!}+\frac{5}{4!}+\cdots$

F. $1-5+\frac{25}{2!}-\frac{125}{3!}+\frac{625}{4!}+\cdots$