

Written Homework for 15.1-15.3

1. Find the critical points and classify them as local maxima, local minima, or saddle points.

A.  $f(x, y) = (x + y)(xy + 1)$ .

B.  $g(x, y) = x + y + \frac{1}{x} + \frac{4}{y}$ . Also determine if the local extrema are global extrema.

2. Sketch contour diagrams of  $g(x, y) = k(x^2 + y^2) - 2xy$  for  $k = -2, -1, 0, 1$ , and  $2$ . You might want to use the MIT Open Courseware posted on our class webpage. Level curves can be accessed under the 'Show' menu. Use the contour diagrams to classify the point  $(0, 0)$  as a local maximum, minimum or saddle point for each value of  $k$ .

3. You are designing a rectangular fish tank that will hold 20 cubic feet of water. The material needed for the bottom will cost \$5 per square foot, the three of the vertical walls will cost \$3 per square foot, and the back wall will cost \$10 per square foot. Find the dimensions that minimize the total cost of materials needed to build the fish tank. Include the total cost.

4. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the constraint.  
 $f(x, y) = 3x - 2y$  subject to  $x^2 + 2y^2 = 44$ .

5. The figure below shows contours labeled with values of  $f(x, y)$  and a constraint  $g(x, y) = c$ . Mark the approximate points at which:

A.  $\nabla f = \lambda \nabla g$

B.  $f$  has a maximum

C.  $f$  has a maximum on the constraint  $g = c$ .

