

Written Homework for 18.2 – 18.4

For problems 1 through 5, use the information below to find the value of the line integral using the indicated technique. Show all work and use proper notation.

$$\vec{F}_1(x, y) = y^2\vec{i} + (2xy + 2y)\vec{j} \quad \vec{F}_2(x, y) = y\vec{i} \quad \vec{F}_3(x, y) = xy\vec{i} + x^2\vec{j}$$

Path A: This path is composed of three pieces.  $C_1$  is the line segment from (2,0) to (0,2).

$C_2$  is the portion of a circle of radius 2 from (0,2) to (-2,0).  $C_3$  is the line segment from (-2,0) to (2,0). Assume motion is counterclockwise.

Path B: This path is a portion of the parabola from (2,4) to (0,0).

1.  $\vec{F}_1(x, y)$  and path B using parameterization (use the given path and orientation).
  2.  $\vec{F}_1(x, y)$  and path B using the Fundamental Theorem.
  3.  $\vec{F}_2(x, y)$  and path A using parameterization. (label the three parts clearly.)
  4.  $\vec{F}_2(x, y)$  and path A using Green's Theorem.
  5.  $\vec{F}_3(x, y)$  and path A using Green's Theorem.
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6. Find the values of  $A$ ,  $B$ , and  $C$  so that  $\vec{F}$  is a gradient field.

$$\vec{F} = \left( \ln(z^2 + 1) - \frac{A}{z^2 + 1} \right) \vec{i} + \left( \ln(z^2 + 1) + By^2 \right) \vec{j} + \left( \frac{C(xz + yz)}{z^2 + 1} + \frac{3xz}{(z^2 + 1)^2} \right) \vec{k}.$$

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For problems 7 through 9, use the information below. Show all work and use proper notation.

$$\vec{F}(x, y) = y\vec{i} + x\vec{j} \quad \vec{G}(x, y) = \frac{y}{x^2 + y^2} \vec{i} - \frac{x}{x^2 + y^2} \vec{j} \quad \vec{H}(x, y) = \frac{x}{\sqrt{x^2 + y^2}} \vec{i} + \frac{y}{\sqrt{x^2 + y^2}} \vec{j}$$

Path: Circle of radius 2 centered at (0,0), traveled counterclockwise.

7. Find a potential function for each field. Include any restrictions on  $x$  and  $y$ .
8. For which fields can you not use Green's Theorem? Why not?
9. Find the value of the line integral for each field.