Math 322-1 Readiness Test, Part A 2007

Homework Partial Solution

II. INTEGRATION

Let $f(x) = 0$ for $0 \le x < 1$,

$f(x) = (x - 1)^2$ for $1 \le x \le 2$.

3. Find the following integrals (as usual, show your work so a reader can figure out how you got your answer):

$\int_0^2 f(x)\,dx$

$\int_0^x f(u)\,du$ for $0 \le x \le 1$.

$\int_1^x f(u)\,du$ for $1 \le x \le 2$.

$\int_0^x f(u)\,du$ for $1 \le x \le 2$.

SOLUTION.

Without showing all work, the first three integrals are:

$\int_0^2 f(x)\,dx = \mathbf{1/3}$.

For $0 \le x \le 1$, $\int_0^x f(u)\,du = \int_0^x 0\,du = \mathbf{0}$, since, when $0 \le x \le 1$, $f(u) = 0$ for $0 \le u \le x$..

For $1 \le x \le 2$, $\int_1^x f(u)\,du = \int_1^x (u-1)^2\,du = \frac{1}{3}(x-1)^3$.

With more details, the last integral, $\int_0^x f(u)\,du$ for $1 \le x \le 2$, is calculated as follows:

Choose any x with $1 \le x \le 2$. Since the function begin integrated (from 0 to x) is defined piecewise in the interval of integration, we have to break up the integral:

$$\int_0^x f(u)\,du = \int_0^1 f(u)\,du + \int_1^x f(u)\,du$$

Now we can use the preceding results:

$$\int_0^x f(u)\,du = 0 + \frac{1}{3}(x-1)^3 = \frac{1}{3}(x-1)^3.$$