

*What has been is what will be,  
and what has been done is what will be done;  
there is nothing new under the sun.  
Is there a thing of which it is said,  
"See, this is new"?  
It has already been,  
in the ages before us. (Ecclesiastes 1: 9-10)*

1. Let  $f$  be the function on  $\mathbb{R}$  defined by  $f(x) = x^2 + x$  for all real numbers  $x$ .
  - a) This problem is essentially the same as a problem which was assigned in Lessons 38 and 38a). The solution method is modeled on Exercises 7.3(a)(b)(c), although this problem (in Lessons 38 and on the exam) is a little more complicated.
  - b) If you understand how to do Exercise 7.7(e) (Lessons 33 and 38), you can do this problem. Also done in class.
- 2.-3. If  $f$  is injective, then ...  $f^{-1}(f(C)) = C$ .  
These are Exercises 7.15(a) and 7.18(a), Lesson 36a and Lesson 39.  
The proofs require you to know the definition of image, the definition of pre-image, the definition of injective, and how to prove that two sets are equal.
4. a) Give the definitions of “finite set” and “denumerable set” ... .  
This requires the basic definitions from Section 8, which were announced as material to be covered on the exam.
  - b) Prove that the set of even natural numbers is denumerable.  
This was the only proof on the exam which was not completely similar to a homework problem; the proof was discussed in class. It requires you to know the definition of denumerable (as in (a)), how to define a function from one set to another set, and how to prove that a function is bijective.
5. Classify the following sets as finite (F), denumerable (D), countable (C), and/or uncountable(U).  
These classifications all were explicitly discussed in class and were announced as material to be covered on the exam.
6. We say that a subset  $S$  of  $\mathbb{R}$  is a **simpleton** iff ...  
The notational issues and proving techniques addressed in this problem
  - have been discussed since early in the semester (January),
  - were in part the subject of a quiz on March 14 whose solutions were posted online,
  - are relevant to the descriptions of circles and lines which have been the subject of homework problems and quizzes,
  - were specifically addressed in the “Wildcat Sets” problems in Lessons 34, 34a, and 34b,
  - were specifically discussed in class in connection with the “Wildcat Sets” problems,
  - are directly related to some of the problems on Project III\*.

\*Recall that in connection with Project III, all of you who submitted a Project turned in a cover sheet which stated “that all of us participated fully and equally in the completion of this assignment”.