Sect. 8:

Do **Exercise 8.4(b)**** completely (even if you have done it before) AFTER solving the following problems*.

1. For each real number a, show that the open intervals $(0, \infty)$ and (a, ∞) are equinumerous.
2. For each real number a, show that the open intervals $(-\infty, -a)$ and (a, ∞) are equinumerous.

* These problems require you to define a function which is a bijection, and **you should also PROVE that it is a bijection**. Be sure that you know how (i) to define a function between two given sets and (ii) to prove that it is a bijection.
See Defining Functions and Function Warnings.

** **READ THE INSTRUCTIONS IN THE BOOK** for part (b). Explain why you can "use part (a)" in Exercise 8.4 only if the intervals in (b) are bounded. (Roughly speaking, a **bounded** interval is an interval which does not have an infinity as an endpoint.) Give a **complete** proof of (b) by **using previous results**, including part (a), and the problems above, and relevant parts of Exercise 8.3. (You are specifically allowed to use Ex. 8.3, even though it has not been assigned as homework. But since you are asked to give a **complete** proof of part (b), do NOT use your previous solution of 8.4(b).)

Sect. 10:

Warmup:
Consider, for each positive integer n, the following statement:
$P(n)$: n students will pass Math 323-1 in the Spring of 2008.
Your instructor tells you that this is true for $n = 5$ and, in fact, that it is true for all n with $n \leq 5$.
What precisely can you conclude about the number of students who will pass the course?

Exercise 10.21
Replace the last sentence of the instructions in the book by the following:
State and prove your result **completely**, using induction where appropriate.
DO THE PROBLEMS IN THIS ORDER: (b)*, (c), (a), and be efficient in your proofs.

* Hint for (b): It may be useful to use the fact that
$n + 1 = n(1 + {}^{1}/_{n})$ (this is a way of expressing $n + 1$ in terms of n).

Exercise 10.30 [using induction for the case of finite subsets of the natural numbers, and then using the result for finite subsets to prove the general case]

Sect. 11:

Warmup:

See Triangle Inequality Practice [link in Lesson 47] and do the problems there.

Exercise 11.6(a)

Use the triangle inequality to prove this.

Sect. 12:

right hand column for each of the following:

Exercises 12.3, 12.4

1. Use the Archimedean Property of the natural numbers, **Theorem 12.10(a)**, to prove that for every real number $x < 1$, there exists a natural number n such that $1 - {}^1/_n > x$.

2. Find the supremum of the set in **Exercise 12.3(f)** and prove your answer using the "Approximation Property" for suprema.

3. Prove your answer for the inf for **Exercises 12.3-4(j)**.
   (You should also be able to prove your answer for sup for **Exercise 12.3(j)**, but it is not required for this assignment.)

Review the result of Lesson 52 and then solve the following:

Exercises 12.8 (last two inequalities only), **12.12(a)**

Sect. 16:

Exercises 16.6(a), (c), (d)*

* In 16.6(d), you can use the fact that the range of the sin function is a subset of the interval [-1, 1].

PRELUDE. A sequence (and a real-valued function in general) is said to be **bounded** iff its range is bounded. In class, we defined the concept of a bounded sequence in terms of the existence of TWO numbers (a lower bound and an upper bound), as follows:

A sequence (s_n) is said to be bounded iff there exist numbers m and M such that for every natural number n,

$$m \leq s_n \leq M.$$

Compare this with the similar definition of a bounded sequence given in Section 16 of the textbook (in terms of just ONE number, M) and prove that the two definitions are equivalent.

Exercise 16.12(a)*

* In 16.12, as usual, prove the universally quantified parts of the definition in the appropriate way. This includes not only the "for all ε " part, but also the "for all n" part.