You are to give a simple geometric description of each of the sets; your description should be elementary and understandable by a high school algebra student familiar with simple coordinate geometry. More information and instructions will be published online.

1. SLIGHTLY MODIFIED FEB 23. Let $m$ be a real number. Consider the subset of $\mathbb{R} \times \mathbb{R}$ given by

$$\{(x, y) \text{ in } \mathbb{R} \times \mathbb{R} : \exists\, b \in \mathbb{R} \ni y = mx + b \}.$$

   For each real number $m$, give a simple alternative description of this set, and prove your answer in the standard way of proving that two sets are equal. See EXAMPLE below.

2. Give a simple alternative description of each of the following sets, and prove your answer in the standard way of proving that two sets are equal. See EXAMPLE below.

   a) $A = \{(x, y) \text{ in } \mathbb{R} \times \mathbb{R} : \forall\, m \in \mathbb{R},\ \forall\, b \in \mathbb{R},\ y = mx + b \}.$

   b) $B = \{(x, y) \text{ in } \mathbb{R} \times \mathbb{R} : \exists\, m > 0 \ni y = mx \}.$

   c) $C = \{(x, y) \text{ in } \mathbb{R} \times \mathbb{R} : \exists\, m \in \mathbb{R} \ni y = mx \}.$

3. DEFINITION. For each real number $m$, we say that a subset $B$ of $\mathbb{R} \times \mathbb{R}$ is **linear of type *m*** iff

$$\forall\, (x, y) \in B,\ y = mx .$$

   Complete the following statement (on your own paper) with a **geometric** description and prove your answer.

   a subset $B$ of $\mathbb{R} \times \mathbb{R}$ is linear of type $m$ iff $B$ is _____

4. Consider the following definitions.

   a) DEFINITION. We say that a subset $B$ of $\mathbb{R} \times \mathbb{R}$ is a **bangle** iff

$$\exists\, m \in \mathbb{R} \ni \forall\, (x, y) \in B,\ y = mx .$$

   b) DEFINITION. We say that a subset $B$ of $\mathbb{R} \times \mathbb{R}$ is a **bead** iff

$$\forall\, (x, y) \in B,\ \exists\, m \in \mathbb{R} \ni y = mx .$$

   For each of these definitions, complete the following statement (on your own paper) with a **geometric** description and prove your answer.

   a subset $B$ of $\mathbb{R} \times \mathbb{R}$ is a **bangle** iff $B$ is _____

   a subset $B$ of $\mathbb{R} \times \mathbb{R}$ is a **bead** iff $B$ is _____

**EXAMPLE** for 1 and 2. SLIGHTLY MODIFIED FEB 23. Consider the subset of $\mathbb{R} \times \mathbb{R}$ given by

$$\{(x, y) \text{ in } \mathbb{R} \times \mathbb{R} : \ \forall\, m \in \mathbb{R}\,,\ x^2 + y^2 \le m^2\ \}.$$

Give a simple alternative description of this set.

**SOLUTION**. This is the set $\{(0, 0)\}$.

Proof (brief). Let $L$ be the set as defined above. Suppose $(x, y)$ is in $L$. Then it is a consequence of Exercise 11.4 that $x^2 + y^2 = 0$ (you should be able to verify this independently; it just happens that Exercise 11.4 is relevant), and from this we can conclude that $x = y = 0$, so that $(x, y) = (0, 0)$. Thus, $L$ is a subset of $\{(0, 0)\}$.
Conversely, if $(x, y) = (0, 0)$, then $x = y = 0$, and so, for every real number $m$, $x^2 + y^2 = 0 \le m^2$. This shows that $\{(0, 0)\}$ is a subset of $L$ and completes the proof that $L = \{(0, 0)\}$.
Thus, the given set is the set $\{(0, 0)\}$.

The purpose of this example is to give you an idea of the kind of answer you might give to one of the questions above. DO NOT COPY THIS SOLUTION AND TRY TO USE IT AS A TEMPLATE for your solutions to the problems above. In particular, Exercise 11.4 is probably irrelevant for the problems you are given above.

Be sure to note that in Problem 1, a number $m$ is given before the definition of the set. This is not the case in Problem 2 or in this Example.

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