Common Errors of Post-Calculus Students

I. Common numerical/logical errors and misconceptions of post-calculus students:
(All variables refer to real numbers unless otherwise specified.)

1. a. If it is true that $x < a$, then it is not true that $x \leq a$.
   b. If $x \neq a$, then it can't be true that $x \leq a$.

2. If one wants to prove that $x \leq a$, and one has proven that $x < a$, one still has to consider the case $x = a$.

3. The following statement is not true: For all real numbers x, $x^2 > -1$.

4. If $y/x > a$, then it always follows that $y > xa$.

5. $|(-x)| = x$.

6. $\sqrt{(x^2)} = x$.

7. $x^2 \geq x$.

8. If $a < b$, then $a^2 < b^2$ (and the converse).

9. If $xu = xv$, then $u = v$.

10. The only numbers in the interval $[0, 2]$ are 0, 1, and 2.
    (More specifically, if one needs to check something for all x in $[0, 2]$, one needs only to check $x = 0, 1, 2$.)

11. The set $\{1/n : n$ is a positive integer $\}$ (i.e., $\{x :$ there exists a positive integer n such that $x = 1/n\}$) is the same as the set $(0, 1]$.

12. "Let m be the smallest number greater than 1."

13. a. To prove that x is an element of the interval $(3, \infty)$, one must prove both $x > 3$ and also $x < \infty$.
    b. If x is an element of the interval $(3, \infty)$, then $x > 3$ and x "goes on forever".

14. a. The equation $x^2 - 4x = y$ has no solutions because there are two variables and only one equation.
    b. One can't solve the equation $x^2 - 4x = y$ for x because y is unknown.
    c. To solve the equation $x^2 - 4x = y$ for x, either
       I don't have a clue if I can't use my calculator, or
       I begin by factoring the left hand side: $x(x - 4) = y$, so $x = y$ or $x - 4 = y$. **…**

15. One can prove a statement by assuming that it is true and showing that it leads to a tautology (such as $1 = 1$). E.g., if we want to prove the statement p, we use the following logic: $p \Rightarrow q \Rightarrow r \Rightarrow$ "1=1". (After completing the "proof", one draws a box around the tautology at the end of the argument and writes, "which is true".)

16. a. If $x^2 < 1$, then $x < \pm 1$. (Is this an error? How?)
    b. The following statement is not true: If $x^2 < 1$, then $x < 1$.

II. Other occasional errors, not as common as those above:

1. If we know that k is a negative number, then $k = -k$. OR: If k is a negative number, then $k = -a$.

2. If $x^2 = 2$, then $x = |\sqrt{2}|$.