

Definition [an imaginary definition, not a definition of a standard mathematical concept].

A number x is called a **gizmo** iff there exists an integer k such that $x = 7k - 3$.

Theorem. If p is a gizmo, then $5p - 4$ is a gadget.

[We imagine that we have a definition of a **gadget**, another imaginary definition.]

Proof. [We imagine that we have already done, or can use in the proof, Exercise 5.4 and Theorem 7, as cited below. Also, **this is not intended as a template for proofs**, merely as an example of a proof.]

Let p be a gizmo. By Exercise 5.4 (previous homework), we know that, since p is a gizmo, then $5p$ is a thingamajig. By Theorem 7 (textbook), we know that a thingamajig minus an even number is a gadget. Since $5p$ is a thingamajig and $4 = 2 \cdot 2$ is an even number, we know that $5p - 4$ is a gadget. This shows that if p is a gizmo, then $5p - 4$ is a gadget.

Comment on Proof.

Let p be a gizmo.

Comment.

This is **a** standard way of starting the proof of universally quantified statements such as,
 “For every gizmo p , ...”

or

“For all p , if p is a gizmo, ...”.

As frequently pointed out in class, there are many other sentences which can be used to start such a proof.

Note that we did NOT start with

“Let $p = 7k - 3$ for some integer k .” [the definition of a gizmo].

There are several reasons (in this course) for not doing this:

1. At this stage of your life, in this section of Math 323, one of the main emphases is on how to start the proof of universally quantified statements. If you are trying to prove something about “all gizmos”, start with an arbitrary gizmo. Then bring in the definition later if needed.
2. As you can see from the rest of this particular proof, the definition of a gizmo is not needed in this particular proof, just properties of gizmos. By introducing the definition immediately, you may overlook this fact, and you are certainly wasting time, paper, and pencil or ink.
3. **It is definitely wrong to start with**
 “Let $p = 7k - 3$ for some integer k , so p is a gizmo.”
 Even if you are going to use the definition of a gizmo for p , the point here is NOT that
 “**IF** $p = 7k - 3$, **THEN** p is a gizmo”
 and this may be entirely irrelevant for the proof. **Be sure you understand!**
4. If you DO need to use the definition of a gizmo, then start in two stages, for example:
 “Let p be a gizmo. Then there exists an integer k such that ...”