

4. We say that a set S of numbers is **semi-absorbent** iff kx is in S whenever x is in S and $k > 0$.
 (c) Prove that the set $\{0\}$ is semi-absorbent.

SOLUTION. According to the definition of semi-absorbent, we want to prove that

$$\forall x \in \{0\}, \forall k > 0, kx \in \{0\} .$$

So we start the proof of this universally quantified statement in the usual way ...

Consider x in $\{0\}$ and $k > 0$. [We want to prove that $kx \in \{0\}$.]

Then $x = 0$, so $kx = 0x = 0$. Since $kx = 0$, $kx \in \{0\}$.

We have proved that if $x \in \{0\}$ and $k > 0$, then $kx \in \{0\}$.

By definition of semi-absorbent, this proves that $\{0\}$ is semi-absorbent.

5. (a) (High school algebra problem) Solve for x : $(4x + 3)(x - 1) = 2$.

SOLUTION. By subtracting 2 from both sides of this equation, and then either factoring, or using the quadratic formula, or completing the square, we find the solution set to be $\{5/4, -1\}$.

- (b) (Math 323 problem) Prove or disprove: There exists a positive number x such that

$$(4x + 3)(x - 1) = 2.$$

SOLUTION. Proof. Let $x = 5/4$. Then x is a positive number and

$$(4x + 3)(x - 1) = (4 \cdot 5/4 + 3)(5/4 - 1) = (8)(1/4) = 2.$$

So, there exists a positive number x (namely, $x = 5/4$) such that $(4x + 3)(x - 1) = 2$, as claimed.

6. (a) **NEN** State in clear, precise, unambiguous English the negation of each of the following statements.

(i) I am useless and hopeless.

(ii) All cats are weak.

(iii) All dogs are big and strong.

COMMENT. As pointed out repeatedly – in the textbook, in homework exercises (**beginning with Lesson 1**), in class, in email discussion of the homework, and in online discussion of the homework – when negating statements with more than one “thing” in the subject, and/or more than one “thing” in the predicate, one must be careful with the use of “not” in the negation. The negation of

“All they are this and that”

is **not**

“All they are not this and that”.

SOLUTION.

(i) The negation of “I am useless and hopeless” is “I am not useless or I am not hopeless”.

(ii) A negation of “All cats are weak” is “Some cats are not weak” or “There is a cat which is not weak” or simply “Not all cats are weak”.

(iii) A negation of “All dogs are big and strong” is “Some dogs are not big or some dogs are not strong”.

(As pointed out in class in connection with “short” and “tall”, the negation of “weak” is not “strong”; the negation of “big” is not “small”.)

(b) **NEN** Write statement (ii) as a universally quantified if-then statement (“for all x , if ... , then ...”).