4. a) Is the relation R in **the previous problem** an equivalence relation on $\{ -1, 0, 1 \}$? **Explain briefly**. Recall: $R = \{ (-1, -1), (0, 0), (1, 1) \}$

SOLUTION. The relation R is in fact the equality relation on $\{ -1, 0, 1 \}$, which is reflexive, symmetric, and transitive, and so is an equivalence relation. In more detail:
Consider x in $\{ -1, 0, 1 \}$. Then the ordered pair (x, x) is in R by inspection, so R is reflexive.
Consider (x, y) in R. Then the reversed ordered pair (y, x) is in R, since all the ordered pairs in R have the form (x, x).
Consider (x, y) and (y, z) in R. Then, by inspection, we must have $x = y = z$, so (x, z) is in R. Thus, R is transitive.
Since R is reflexive, symmetric, and transitive, it is an equivalence relation.

b) If the answer is "yes", find the equivalence class of 0 and explain completely.
If the answer is "no", prove your answer.

SOLUTION. The equivalence class of 0 is the set of all elements x of $\{ -1, 0, 1 \}$ which satisfy $x R 0$, i.e., $(x, 0)$ is in R. Clearly, the only such element is 0, so the equivalence class of 0 is $\{0\}$.

5. a) Give a correct definition of what it means for an integer to be an **odd number**:
"n is an odd number iff ... " YOU WILL USE THIS DEFINITION ON PART (b).

SOLUTION. n is an odd number iff the exists an integer k such that $n = 2k + 1$.

b) Prove or disprove, using your definition: If p and q are odd numbers, then pq is an odd number.

SOLUTION. Suppose p and q are odd numbers. Then, by definition of odd, we can find integers a and b such that $p = 2a + 1$ and $q = 2b + 1$. So $pq = (2a + 1)(2b + 1) = 2(2ab + a + b) + 1$.
Since $2ab + a + b$ is an integer, this proves that pq is odd, as desired.

c) Prove or disprove: If p is an odd number, then p^2 is an odd number.

SOLUTION. Suppose p is an odd number. By part (b), pp is odd, so p^2 is odd, as desired.

6. Decide whether the following relations are transitive and prove your answers (or explain completely):
a) $A = \{(0,1)\}$ **b)** $B = \{(0,1), (1,0)\}$

SOLUTION. The relation A **is transitive** because there are no "matching" ordered pairs of the form (x, y) and (y, z) to violate the transitivity condition. The relation B **is NOT transitive** because, while the matching ordered pairs $(0, 1)$ and $(1, 0)$ are in the relation, the "offspring" $(0, 0)$ is not.

7. a) Consider the set $\mathbb{R} \times \mathbb{R}$. To make sure you what know set we're talking about, choose a **specific** element in this set and write it down.

SOLUTION. For example $(0, 0)$.

b) Let A be a nonempty subset of $\mathbb{R} \times \mathbb{R}$. On this set A, let R be the relation defined as follows:
For points p and q in A, pRq iff there is a piecewise continuous path in A which joins p to q.
Suppose you want to prove that this relation R is reflexive on A.
How would you start the actual proof that R is reflexive on A?

SOLUTION. Consider an arbitrary element p of A.
Or, since A is a subset of $\mathbb{R} \times \mathbb{R}$, consider an arbitrary element (x, y) of A.