Math 110/112 Final Exam Study Aid

Spring 2004

Note. This study aid is intended to help you review for the final exam. It covers the primary concepts in the course. Although the final exam will be similar to the study aid, it will not be identical to it. You should also review tests, notes, study aids and homework given during the semester. Formulas are given on the last page.

1. Which of the following equations determine y as a function of x?

   (1) $3x + 2y^3 = 10$ (2) $\sqrt{x-1} + y = 8$ (3) $2x - y^2 - 7 = 0$
   (4) $3x^2 - xy = 1$

   (A) All of them (B) 1 and 3 only (C) 1, 2 and 4 only
   (D) 1 and 2 only (E) 1 and 4 only

2. Which of the following tables determine y as a function of x?

   (1)

x	y
4	1
5	3
7	3
8	4

   (2)

x	y
4	1
5	2
6	4
4	5

   (3)

x	y
1	8
2	4
3	2
2	4

   (A) 1 only (B) 2 only (C) 3 only
   (D) 1 and 3 only (E) 2 and 3 only

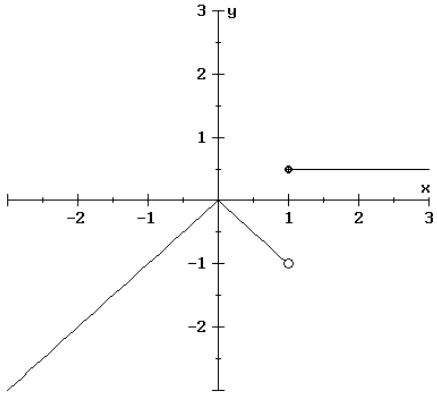
3. If $f(x) = 2^x + 3x$, find $f(-2)$.

   (A) $-\frac{23}{4}$ (B) -10 (C) $-\frac{5}{4}$ (D) -4 (E) None of these

4. If $g(x) = \begin{cases} 1 - 3x & \text{for } x < -1 \\ 3 - x^2 & \text{for } x \geq -1 \end{cases}$, what is $g(-3)$?

   (A) 12 (B) 10 (C) -8 (D) -6 (E) None of these

5. Which of the following is the piecewise equation for the graph below?



(A) $f(x) = \begin{cases} -x^2 \text{ for } x < 1 \\ 1/2 \text{ for } x \geq 1 \end{cases}$ (B) $f(x) = \begin{cases} 1/2 \text{ for } x > 1 \\ |x| \text{ for } x < 1 \end{cases}$

(C) $f(x) = \begin{cases} -|x| \text{ for } x < 1 \\ 1/2 \text{ for } x \geq 1 \end{cases}$ (D) $f(x) = \begin{cases} 1/2 \text{ for } x > 1 \\ -|x| \text{ for } x < 1 \end{cases}$

(E) None of these

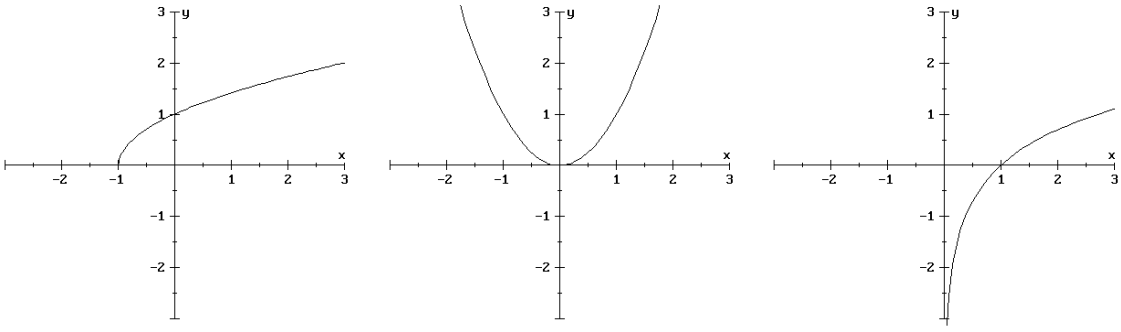
6. If $f(x) = 3x^2 - 2$, find $f(b-2)$.

(A) $3b^2 - 12b + 12$ (B) $3b^2 - 8$ (C) $3b^2 - 4b + 2$
(D) $3b^2 - 4$ (E) None of these

7. What is the implied DOMAIN of the function $f(x) = 12 - \sqrt{108 - 3x}$?

(A) $(-\infty, 36]$ (B) $[-108, 108]$ (C) $[36, \infty)$
(D) All real numbers except 36 (E) None of these

8. Which of the following have a RANGE of $[0, \infty)$?



(1) (2) (3)

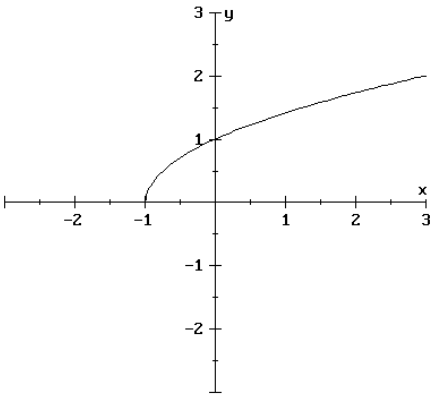
(A) 2 only (B) 1 and 3 only (C) 1 only
(D) 1 and 2 only (E) All of them

9. Which of the following has a domain of all real numbers except 18?

(1) $f(x) = \sqrt{x - 18}$ (2) $g(x) = \dfrac{2x}{x - 18}$ (3) $h(x) = \dfrac{1}{x^2 - 324}$

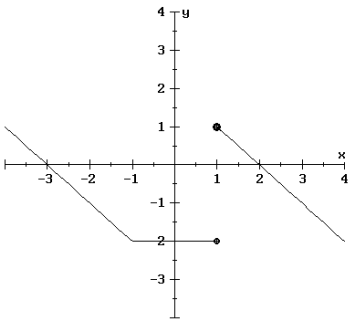
(A) 1 only (B) 2 only (C) 2 and 3 only (D) 1 and 2 only (E) All of them

10. What is the DOMAIN of the function represented by the graph below?

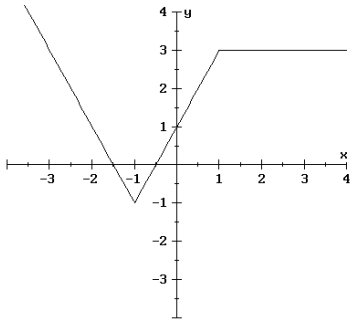


(A) $(-\infty, \infty)$ (B) $[0, \infty)$ (C) $[-1, \infty)$ (D) $(-\infty, -1]$ (E) None of these

11. Which of the following graphs represent y as a function of x?



(1)



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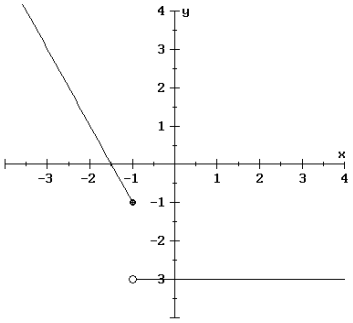

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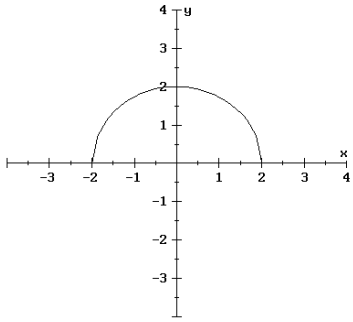


(4)

(A) All of them (B) None of them (C) 2, 3, and 4 only
(D) 2 and 3 only (E) 1 only

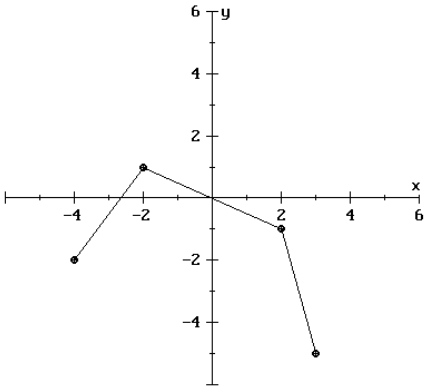
12. Express the area of a rectangle AS A FUNCTION OF ITS WIDTH if the width is 25% of its length. Let L and W represent length and width, respectively.

(A) $A = (.25W)(W)$ (B) $A = (.75W)(W)$ (C) $A = (4W)(W)$
(D) $A = 4LW$ (E) None of these

13. Find the zeros of the function $g(x) = \dfrac{3x^2 - 5x + 2}{\sqrt{x+2}}$

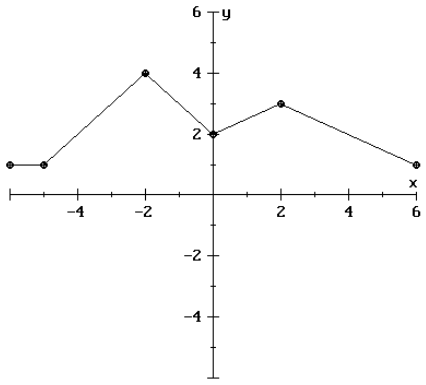
(A) $\frac{2}{3}$, 1 and -2 only (B) $\frac{1}{3}$, 2, and -2 only (C) $-\frac{2}{3}$ and -1 only
(D) $\frac{1}{3}$ and 2 only (E) $\frac{2}{3}$ and 1 only

14. What is the RANGE of the function graphed below?



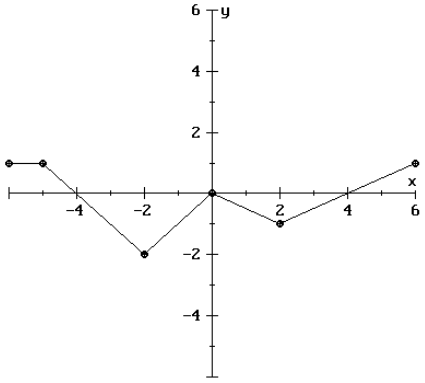
(A) $[-4, 3]$ (B) $[-5, -2]$ (C) $[-2, 3]$ (D) $[-5, 1]$ (E) None of these

15. The graph of $y = f(x)$ is shown below



This is the graph of $y = f(x)$

The graph below is a transformation of the graph of $y = f(x)$. Which of the following is the formula for the function graphed below?



(A) $y = f(-x) - 2$ (B) $y = -f(x) - 2$ (C) $y = -f(x + 2)$
(D) $y = -f(x) + 2$ (E) $y = f(-x + 2)$

16. The relation that vertically shrinks the graph of $y = \sqrt{x}$ and shifts the graph up twenty units is:

(A) $y = \frac{5}{3}\sqrt{x} + 20$ (B) $y = \sqrt{\frac{7}{2}x + 20}$ (C) $y = \frac{3}{4}\sqrt{x} + 20$
(D) $y = 2\sqrt{x + 20}$ (E) None of these

17. If $(5, -6)$ is a point on the graph of $y = g(x)$, which of the following must be a point on the graph of $y = -g(x) + 1$?

(A) $(-5, -6)$ (B) $(-5, 7)$ (C) $(5, 6)$ (D) $(5, 7)$ (E) $(-6, 6)$

18. You can get the graph of $y = -f(2x)$ by transforming the graph of $y = f(x)$ in the following way:

(A) Shrink horizontally and reflect across the x-axis
(B) Shrink horizontally and reflect across the y-axis
(C) Stretch vertically and reflect across the x-axis
(D) Stretch vertically and reflect across the y-axis
(E) None of these

19. Given the values for $f(x)$ and $w(x)$ below:

x	0	1	2	3	4	5	6	7
$f(x)$	0	.3	2	2.3	3	3.3	4	4.3

x	-2	-1	0	1	2	3	4	5
$w(x)$	0	.3	2	2.3	3	3.3	4	4.3

Find an equation for $w(x)$ relative to $f(x)$.

(A) $w(x) = f(x) - 2$ (B) $w(x) = f(x + 2)$ (C) $w(x) = f(x - 2)$
(D) $w(x) = f(x) + 2$ (E) None of these

Consider the following situation: In 1990 Joe bought a BMW for \$40,000; it depreciates \$4,000 each year. Tom bought a classic '57 Chevy for \$10,000 and it appreciates \$1,000 each year. Use this information to answer the next TWO questions.

20. Write a system of equations for the values (V) of these cars in terms of the number of years (t). [$t = 0$ corresponds to the year 1990]

(A) BMW : $V = 40,000 - 4000t$
Chevy: $V = 10,000 + 1000t$

(B) BMW: $V = 40,000 + 4000t$
Chevy: $V = 10,000 - 1000t$

(C) BMW : $V = 40,000 - t$
Chevy: $V = 10,000 + t$

(D) BMW: $V = t + 40,000$
Chevy: $V = t + 10,000$

(E) None of these

21. How many years must pass before the values of the cars are equal? The answer is a number:

(A) between 1 and 5 (B) between 5 and 10 (C) more than 10
(D) values are never equal (E) None of these

22. Search and Rescue teams are used in remote areas in the West to find lost people. The team's chance of finding an individual depends on the distance by which team members are separated. The percentage found for various separation distances is shown in the table below.

Separation distance (ft)	Percent found
20	90
40	80
60	70
80	60
100	50

Find an equation to express the percent found, P, as a function of the separation distance, d,of the team members.

(A) $P = .5d + 100$ (B) $P = 290 - .5d$ (C) $P = 100 - .5d$
(D) $P = 90 - 5d$ (E) $P = 2d + 50$

23. Find the equation of the line passing through the points $(2, 1)$ and $(4, 7)$.

The slope and y-intercept are:

	slope:	y-intercept
(A)	3	(0,7)
(B)	1/3	(0,1/3)
(C)	1/3	(0,5/3)
(D)	3	(0,−5)
(E)	None of these	

24. What is the equation of the line PERPENDICULAR to $3y + 2x - 3 = 0$, passing through the point $(4, -1)$?

(A) $2y - 3x + 14 = 0$ (B) $2y + 3x - 10 = 0$ (C) $3y + 2x - 5 = 0$
(D) $3y + 2x + 11 = 0$ (E) None of these

25. Which of the following lines is PARALLEL to $3x - 4y = 7$?

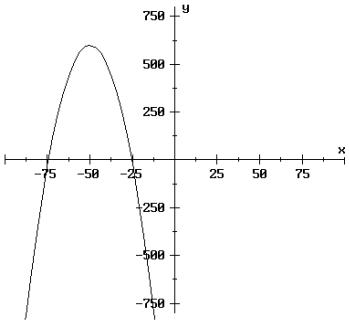
(A) $y = 3x - 7$ (B) $y = -\frac{3}{4}x + 8$ (C) $y = -\frac{4}{3}x - 2$
(D) $y = -\frac{4}{3}x - 3$ (E) None of these

26. What is the degree and leading coefficient of the polynomial $f(x) = -3x^2 + 6x^4 - 4x^5 + 7$?

(A) degree is 2, leading coefficient is -3
(B) degree is 11, leading coefficient is -4
(C) degree is 5, leading coefficient is -4
(D) degree is 5, leading coefficient is 4
(E) None of these

27. Which of the following graphs best represents the quadratic function $f(x) = 600 - (x-50)^2$?

(A)



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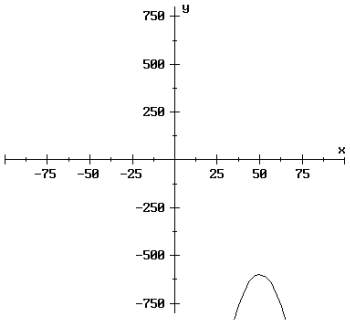
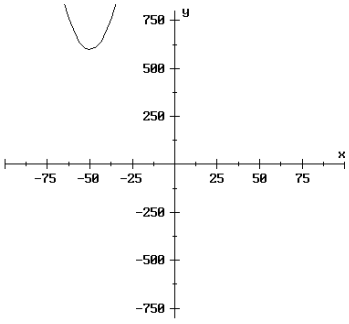
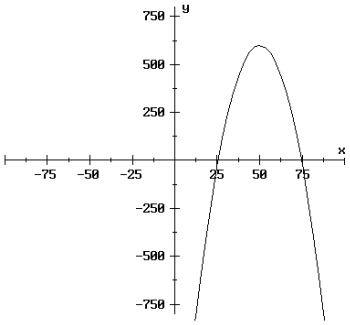


(C)



(D)



28. For the function $f(x) = -x^2 + 6x + 62$, which of the following statements is/are true?

(1) the x-coordinate of the vertex is -3
(2) $f(x)$ has a maximum value of 71
(3) one of the x-intercepts is $\left(3 + \sqrt{71}, 0\right)$

(A) 1 and 2 only (B) 1 and 3 only (C) 2 and 3 only
(D) 2 only (E) 1 only

29. Write $f(x) = 3x^2 + 60x - 1$ in vertex form. The SUM of the x and y coordinates of the vertex is:

(A) -473 (B) 365 (C) -365 (D) 473 (E) None of these

30. Find the vertex of the quadratic function $y = x^2 + 2ax + b$. The y-coordinate of the vertex is:

(A) b (B) $-b$ (C) $\dfrac{b^2}{4a^2} - b$ (D) $-a^2 + b$ (E) $3a^2 + b$

31. Find the vertex of the quadratic function $f(x) = 3x^2 + 7x + 4$.

The vertex is located:

(A) Above the x-axis (B) Below the x-axis
(C) At the origin (D) On the x-axis but not at the origin
(E) None of these

32. Find the vertex of the quadratic function $f(t) = \dfrac{4}{7}t^2 - \dfrac{16}{7}t + 3$.

The y-coordinate of the vertex is:

(A) $\dfrac{1}{2}$ (B) $\dfrac{5}{7}$ (C) $\dfrac{6}{7}$ (D) 1 (E) None of these

33. Find the equation of the parabola that has a vertex of $(3, -1)$ and passes through the point $(4, 1)$. The coefficient of x^2 is a number:

(A) between -2 and $-.5$ (B) between $-.5$ and 1.5
(C) between 1.5 and 3 (D) between 3 and 4.5
(E) None of these

34. Which of the following has two x-intercepts?

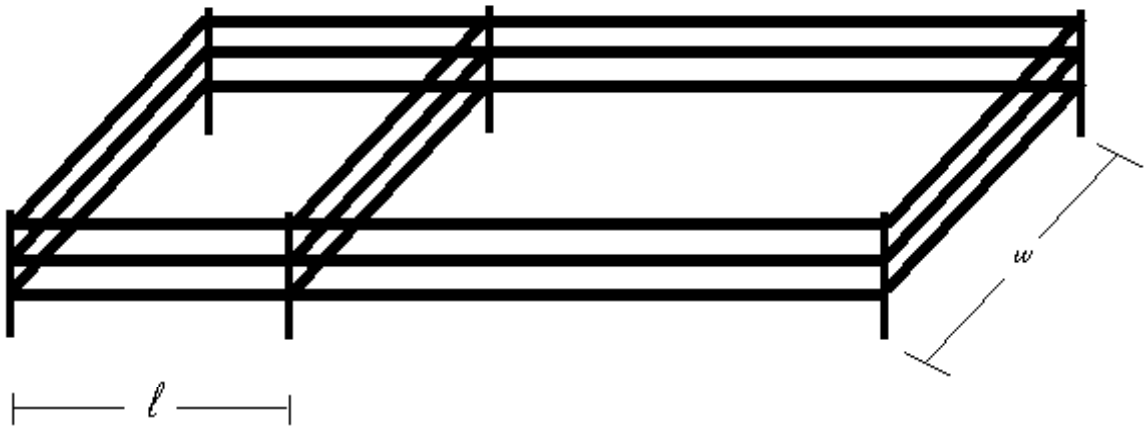
(1) $f(x) = x^2 - 40$ (2) $g(x) = -x^2 + 60x - 903$
(3) $h(x) = -(x + 125)^2 + 1$

(A) 1 only (B) 2 and 3 only (C) 2 only
(D) 1 and 3 only (E) None of these

35. A horticulturist has determined that the number of inches a young oak tree grows in one year is a function of the annual rainfall, r, given by $g(r) = -.01r^2 + .1r + 2$. What is the maximum number of inches a young oak can grow in a year? The maximum number of inches is:

(A) less than 1 (B) between 1 and 2 (C) between 2 and 3
(D) between 3 and 4 (E) between 4 and 5

36. A rancher wishes to enclose two adjacent rectangular corrals such that the right-hand corral has twice the length of the left hand (see diagram). She has 900 feet of fencing. What is the maximum area she can enclose?



(A) 22,500 sq. ft. (B) 50,625 sq. ft. (C) 62,500 sq. ft.
(D) 67,500 sq. ft. (E) None of these

37. For the years 1975 through 1990, the average price, p (in dollars per million British thermal units), of fuel used to generate electricity in the U.S. can be modeled by the function

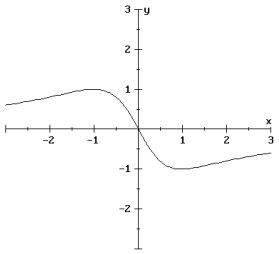
$$p(t) = -.021t^2 + .50t - 1.04$$

where t is time in years since 1970. Estimate the maximum average price, p, of fuel used according to this model.

(A) \$11.9 per million Btu (B) \$1.94 per million Btu (C) \$2.1 per billion Btu
(D) \$10.2 per million Btu (E) None of these

38. Which of the following functions are neither even nor odd?

(1) $f(x) = |15x|$ (2) $f(x) = \dfrac{2x}{x^2+15}$ (3) $f(x) = 217 - 3x - x^3$
(4)



(A) All of them (B) None of them (C) 2 and 3 only (D) 3 only (E) 2 only

For the following TWO questions, use the partial table of values for the function $y = f(x)$ shown below:

x	$f(x)$
-3	9
-2	
-1	-6
1	
2	0
3	

39. Complete the table above so that $f(x)$ is an ODD function.

    The missing values, in order, are:

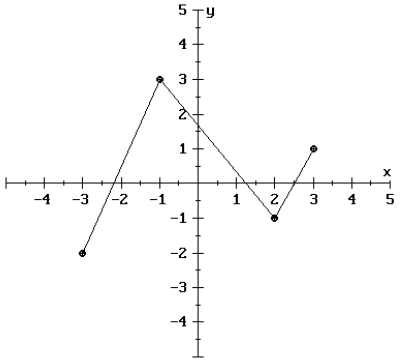
    (A) $0, 6, -9$ (B) $0, -6, 9$ (C) $0, -\frac{1}{6}, \frac{1}{9}$
    (D) $0, \frac{1}{6}, -\frac{1}{9}$ (E) Cannot be determined

40. Complete the table above so that $f(x)$ is an EVEN function.

    The missing values, in order, are:

    (A) $0, 6, -9$ (B) $0, -6, 9$ (C) $0, -\frac{1}{6}, \frac{1}{9}$
    (D) $0, \frac{1}{6}, -\frac{1}{9}$ (E) Cannot be determined

41. Determine the intervals over which the function graphed below is increasing and decreasing.



    Which of the following statements is TRUE about this function?

    (A) The function is **increasing** on $(-2, 3)$; **decreasing** on $(-1, 3)$
    (B) The function is **increasing** on $(-3, -1)$ and $(2, 3)$; **decreasing** on $(-1, 2)$
    (C) The function is **increasing** on $(-1, 2)$; **decreasing** on $(-3, -1)$ and $(2, 3)$
    (D) The function is **increasing** on $(-1, 1)$; **decreasing** on $(-1, 2)$
    (E) The function is **increasing** on $(-3, 0)$ and $(1, 3)$; **decreasing** on $(0, 1)$

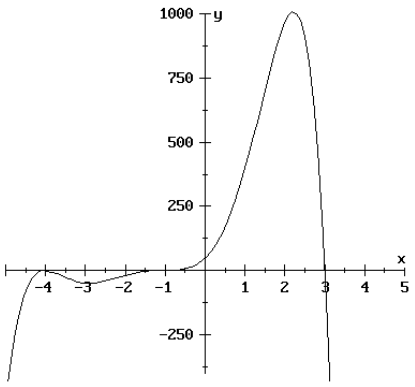
42. The number of miles per gallon, M, for an experimental engine is given by

$$M = \frac{2000x}{1000 + x^2} + 5$$

where x is the speed of the car in miles per hour, $10 \leq x \leq 60$. Using your calculator, determine the speed that yields the greatest number of miles per gallon.

(A) about 60 mph (B) about 10 mph (C) about 37 mph
(D) about 32 mph (E) There is no maximum

43. Which of the following could be the equation of the polynomial $P(x)$ graphed below?



(A) $P(x) = (x-4)^2(x+3)^2(x-1)$ (B) $P(x) = (x+4)^3(x-3)(x+1)$
(C) $P(x) = -(x+4)^2(x-3)(x+1)^3$ (D) $P(x) = (x-4)^2(x+3)(x+1)$
(E) $P(x) = -(x-4)^2(x+3)(x-1)^3$

44. For the graph of $y = -3x^4 + 37x^3 + 28x^2 + 42$, which of the following is correct?

(A) $y \longrightarrow -\infty$ as $x \longrightarrow -\infty$
$y \longrightarrow \infty$ as $x \longrightarrow \infty$

(B) $y \longrightarrow -\infty$ as $x \longrightarrow -\infty$
$y \longrightarrow -\infty$ as $x \longrightarrow \infty$

(C) $y \longrightarrow \infty$ as $x \longrightarrow -\infty$
$y \longrightarrow \infty$ as $x \longrightarrow \infty$

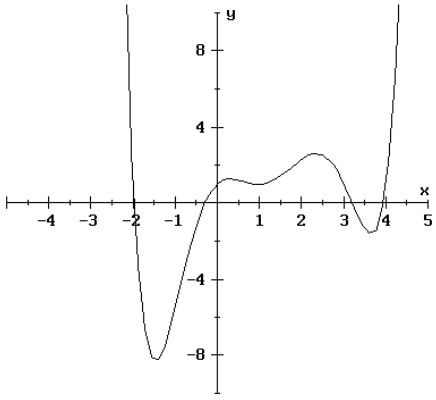
(D) $y \longrightarrow \infty$ as $x \longrightarrow -\infty$
$y \longrightarrow -\infty$ as $x \longrightarrow \infty$

45. For the function $f(x) = -3(x-a)^5(x+b)(x-c)^2$, where a, b and c are positive real numbers, which of the following is/are correct?

(1) $f(x)$ is a polynomial of degree 5
(2) $f(x)$ has a zero at $x = b$
(3) $f(x)$ has an x-intercept at $(c, 0)$
(4) $y \longrightarrow -\infty$ as $x \longrightarrow -\infty$ and $y \longrightarrow -\infty$ as $x \longrightarrow \infty$

(A) 1 and 2 only (B) 2 and 4 only (C) 1 and 3 only (D) 3 and 4 only

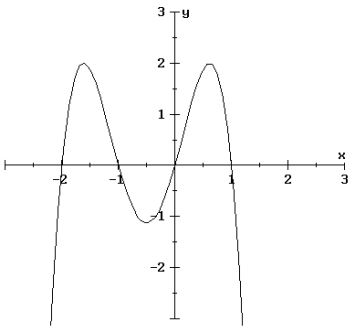
46. Consider the polynomial function below. Which of the following statement(s) is/are true about this polynomial?



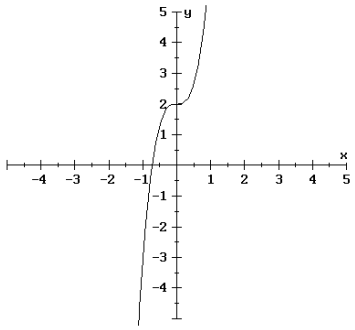
(1) This polynomial could have degree 6
(2) This polynomial could have degree 4
(3) This polynomial could have degree 8

(A) 1 and 2 only (B) 1 and 3 only (C) 1 only
(D) 2 only (E) 2 and 3 only

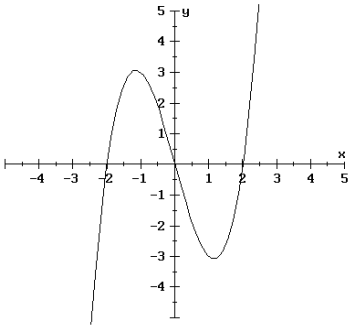
47. Which of the following could be the graph of a polynomial of odd degree?



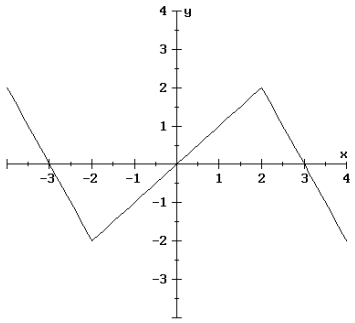
(1)



(2)



(3)



(4)

(A) All of them (B) None of them (C) 1, 2 and 3 only
(D) 1, 2 and 4 only (E) 2 and 3 only

48. Find all the real zeros of $f(x) = x^3 + 5x^2 + 7x + 2$. The LARGEST real zero is:

(A) $\dfrac{-3+\sqrt{5}}{2}$ (B) -0.5 (C) $\dfrac{-3+\sqrt{13}}{2}$

(D) $\dfrac{-3+\sqrt{7}}{2}$ (E) -2

49. Which of the following statements is/are equivalent to:
"$x + 3$ is a factor of the polynomial $f(x)$"?

(1) $x = 3$ is a solution of $f(x) = 0$

(2) $x = -3$ is a zero of $f(x)$

(3) $(-3, 0)$ is an x-intercept of $f(x)$

(A) 1 only (B) 2 only (C) 3 only
(D) 1 and 3 only (E) 2 and 3 only

50. Find all real zeros of $f(x) = 3x^4 - 36x^2 + 60$. The SMALLEST real zero of $f(x)$ is:

(A) $-\sqrt{2}$ (B) $\frac{7}{5}$ (C) $-\frac{16}{5}$ (D) $-\sqrt{10}$ (E) $\sqrt{2}$

51. For which of the following functions does $y \longrightarrow \infty$ as $x \longrightarrow -\infty$?

(1) $y = 4x^3 - 3x$

(2) $y = 2x^4 - x + 10$

(3) $y = -x^6 + 3x^5 - 2$

(A) None of them (B) 2 only (C) 2 and 3 only
(D) 1 and 3 only (E) 3 only

52. Find the value of 'b' so that $x + 2$ is a factor of $P(x) = x^4 - (b + 1)x^2 - 5bx - 9b$.

(A) $b = -2$ (B) $b = 4$ (C) $b = 2$ (D) $b = -4$ (E) None of these

53. What is the remainder when $5x^3 - 6x^2 + 3$ is divided by $x^2 - x + 4$?

(A) $-21x + 7$ (B) 7 (C) $21x - 7$
(D) $-14x$ (E) None of these

54. What is the remainder when $p(x) = x^4 + x^3 - x^2 - 2$ is divided by $x + 3$?

(A) -26 (B) 43 (C) -17 (D) 0 (E) None of these

55. Factor $T(x) = x^3 + x^2 + 13x + 30$. One of the factors is:

(A) $x + 3$ (B) $x - 5$ (C) $x + 2$ (D) $x - 10$ (E) None of these

56. $x = 3$ is a solution of the equation $18x^3 - 111x^2 + 161x + 30 = 0$. What are the other real solutions of equation? The SMALLEST real solution is:

(A) -3.2 (B) $16/5$ (C) $-1/6$ (D) $-1/5$ (E) 0

57. Find a polynomial of lowest degree having zeros $-2, 1, 0$ (a zero of multiplicity 2), and -4 (a zero of multiplicity 3).

(A) $f(x) = x^2(x + 2)(x - 1)(x + 4)^3$ (B) $f(x) = x^2(x - 2)(x + 1)(x - 4)^3$
(C) $f(x) = 2x(x + 2)(x - 1)(x + 4)^3$ (D) $f(x) = (x + 2)(x - 1)(x + 4)^3$
(E) None of these

58. Are the zeros of $p(x) = x^4 + (3/4)x^3 - (1/2)x^2 + (1/4)x + 5$ the same as the zeros of $q(x) = 4x^4 + 3x^3 - 2x^2 + x + 20$?

(A) No (B) Yes

59. Find a polynomial of lowest degree that has $-3, -\sqrt{2}$, and $\sqrt{2}$ as its zeros.

(A) $x^3 + 3x^2 - 2x - 6$ (B) $x^2 + (-3 - \sqrt{2})x - 6$
(C) $x^3 + 3x^2 - 4x - 12$ (D) $x^2 - 2\sqrt{2x} + 3$
(E) None of these

60. Which of the following MUST be true?

(1) A polynomial of degree 4 has four unique zeros.
(2) A polynomial of degree 5 has at least 1 real zero.
(3) A polynomial of degree 2 has at least 1 rational zero.

(A) 1 only (B) 2 only (C) 3 only
(D) 1 and 2 only (E) 1 and 3 only

61. Find all rational zeros of $p(x) = x^4 - 4x^3 + 2x^2 + 5x - 2$. The SUM of all the RATIONAL zeros is:

(A) 1 (B) 2 (C) -1 (D) 0 (E) There are no rational zeros

62. Find all the asymptotes of the rational function $y = \dfrac{2x^2 + 1}{2x + 3}$

(A) $x = -\dfrac{3}{2}, \quad y = x - \dfrac{3}{2}$ (B) $x = -\dfrac{3}{2}, \quad y = 1$ (C) $x = \dfrac{3}{2}, \quad y = x + 3$

(D) $x = \dfrac{3}{2}, \quad y = 1$ (E) $x = -\dfrac{3}{2}, \quad y = x + \dfrac{1}{2}$

63. Find the vertical asymptote(s), if any, for $f(x) = \dfrac{x+1}{4x^2-1}$.

(A) $x = \dfrac{1}{4}$ (B) $x = \dfrac{1}{2}$ (C) $x = -\dfrac{1}{2}, x = \dfrac{1}{2}$

(D) $x = -\dfrac{1}{4}, x = \dfrac{1}{4}$ (E) $x = -1$

64. Determine the behavior of the function $f(x) = \dfrac{2x+1}{1-18x}$ as $x \longrightarrow \infty$.

(A) $y \longrightarrow -\dfrac{1}{9}$ (B) $y \longrightarrow 2$ (C) $y \longrightarrow 0$ (D) $y \longrightarrow \infty$
(E) $y \longrightarrow \dfrac{1}{18}$

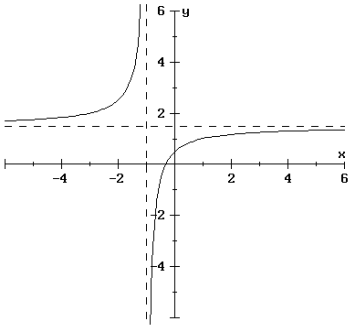
65. Which of the following rational functions has no vertical asymptote?

(1) $y = \dfrac{2}{x^2-5}$ (2) $y = \dfrac{5x-4}{x^2+1}$ (3) $y = \dfrac{7x^2}{-x^2+3}$

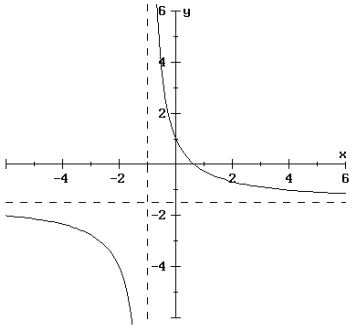
(A) 1 and 3 only (B) 1 and 2 only (C) 2 and 3 only
(D) 2 only (E) 1 only

66. Which of the following is the graph of $y = \dfrac{2x+1}{2x-3}$

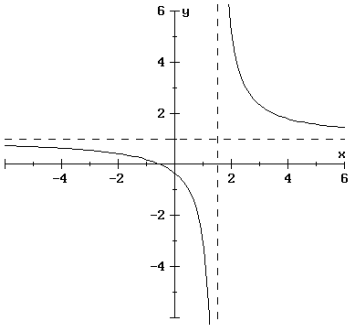
(A)



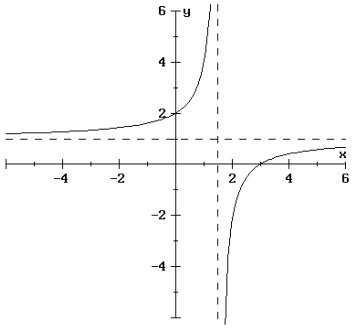
(B)



(C)



(D)



(E) None of these

67. Which of the following is/are TRUE about the function $R(x) = \dfrac{15}{x^2 - x - 6}$?

(1) $x = 15$ is a zero of $R(x)$.
(2) $R(x)$ has a y-intercept of $\left(0, -\frac{5}{2}\right)$.
(3) $R(x)$ has no real zeros.

(A) 1 only (B) 2 only (C) 3 only
(D) 2 and 3 only (E) 1 and 2 only

68. Which of the following statement(s) is/are true about the function $f(x) = \dfrac{6x + 1 + .05x^2}{x - 2}$?

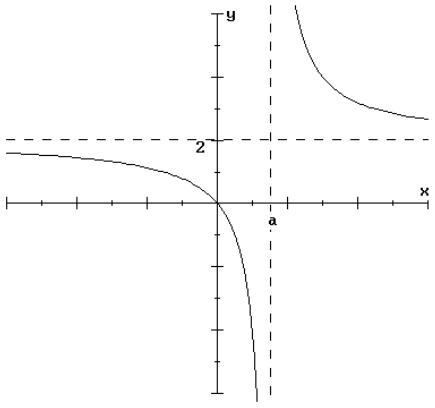
(1) $y \longrightarrow 6$ as $x \longrightarrow \infty$

(2) $f(x)$ has exactly one x-intercept

(3) $f(x)$ has a slant asymptote

(A) 1 and 2 only (B) 2 and 3 only (C) 2 only (D) 3 only (E) 1 only

69. Find a formula for the rational function shown below.



(A) $y = \dfrac{2}{x - a}$ (B) $y = \dfrac{2x}{x + a}$

(C) $y = \dfrac{2x}{x - a}$ (D) $y = \dfrac{x + 2}{x - a}$

(E) $y = \dfrac{x - a}{2x}$

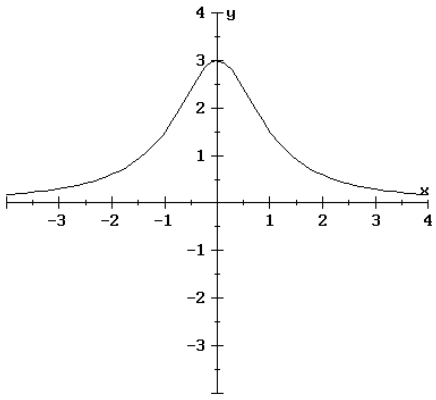
70. Which of the following is the equation of a rational function whose graph:

(i) Has a vertical asymptote at $x = 20$ and $x = -20$
(ii) Has an x-intercept $(1, 0)$
(iii) Has a horizontal asymptote at $y = 0$

(A) $f(x) = \dfrac{x^2 - 400}{x + 1}$ (B) $f(x) = \dfrac{x + 1}{x^2 - 20}$ (C) $f(x) = \dfrac{x - 1}{x^2 - 400}$

(D) $f(x) = \dfrac{x + 1}{x^2 - 20}$ (E) None of these

71. If $y = \dfrac{3}{f(x)}$ is the equation for the graph below, which of the following are NOT possible expressions for $f(x)$?



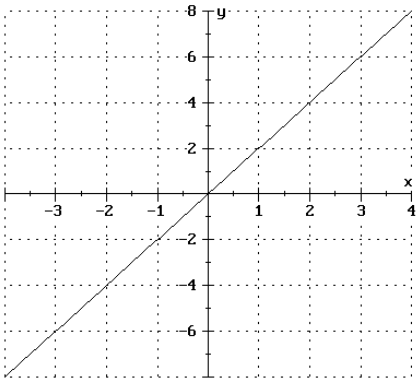
(1) $x + 1$ (2) $x^2 + 1$ (3) $x^2 - 4$ (4) x

(A) 2 only (B) 1 and 4 only (C) 2 and 3 only
(D) 1, 3 and 4 only (E) All are possible

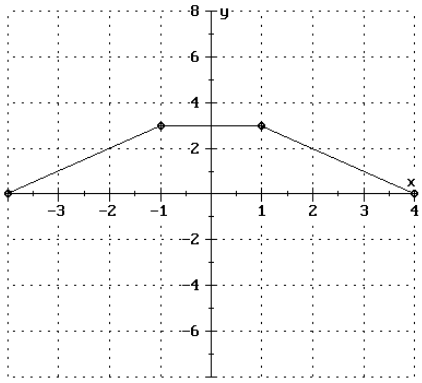
72. Given $f(x) = 4x + 1$ and $g(x) = |5x + 2|$, find $\left(\dfrac{f}{g}\right)(-4)$.

(A) $-\dfrac{15}{22}$ (B) $\dfrac{17}{22}$ (C) $-\dfrac{5}{6}$ (D) -4 (E) None of these

Use the graphs below to answer the next TWO questions.



This is the graph of $y = f(x)$



This is the graph of $y = g(x)$

73. Using the graphs above, find $(f - g)(3)$.

(A) 3 (B) 5 (C) 15 (D) 6 (E) Not enough information

74. Using the graphs above, find $(f \circ g)(1)$.

(A) 6 (B) 2 (C) 1 (D) 3 (E) None of these

75. Given $f(x) = \log_4(-8x)$ and $h(x) = x - 3$, find $(h \circ f)(-2)$.

(A) 1 (B) 10 (C) -1 (D) 0 (E) None of these

76. Given $f(x) = 3 + 2x$ and $h(x) = \sqrt{x}$, find $(f \circ h)(x)$.

(A) $\sqrt{3 + 2x}$ (B) $3 + 2\sqrt{x}$ (C) $\sqrt{3} + \sqrt{2x}$
(D) $3\sqrt{x} + 2x\sqrt{x}$ (E) None of these

77. Suppose that $h(x) = f(g(x))$. If $h(x) = \dfrac{1}{(x+3)^2}$, which of the following is NOT a possible choice for $f(x)$ and $g(x)$?

(A) $f(x) = \dfrac{1}{x^2}$ and $g(x) = x + 3$ (B) $f(x) = \dfrac{1}{x}$ and $g(x) = (x+3)^2$
(C) $f(x) = x + 3$ and $g(x) = \dfrac{1}{x^2}$ (D) $f(x) = x$ and $g(x) = (x+3)^{-2}$

78. If $f(x)$ is a one-to-one function, and $f(2) = 7$, then which of the following CANNOT be true?

(A) $f(7) = 2$ (B) $f^{-1}(7) = 2$ (C) $f^{-1}(5) = 3$
(D) $f(-2) = 4$ (E) $f(-2) = 7$

79. If $G(x) = \sqrt[3]{4-x}$, what is $G^{-1}(x)$?

(A) $G^{-1}(x) = \dfrac{1}{\sqrt[3]{4-x}}$ (B) $G^{-1}(x) = -\sqrt[3]{4-x}$ (C) $G^{-1}(x) = 4 - x^3$
(D) $G^{-1}(x) = x^3 + 4$ (E) None of these

80. Consider the functions $f(x)$ and $g(x)$ represented by the tables shown below.

x	$f(x)$
2	4
3	3
4	6
5	5

x	$g(x)$
2	4
3	5
4	3
5	4

Which, if either, of these functions is one-to-one?

(A) both $f(x)$and $g(x)$ (B) $g(x)$only (C) $f(x)$only
(D) neither $f(x)$nor $g(x)$

81. If the point (2, 6) is on the graph of $f(x)$, which of the following points must be on the graph of $f^{-1}(x)$?

(A) $(-2,-6)$ (B) $\left(2,\frac{1}{6}\right)$ (C) $(6,2)$ (D) $(2,-6)$ (E) None of these

82. If $h(x) = 20x - 62$, what is $h(h^{-1}(6))$?

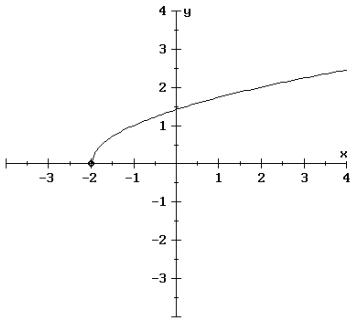
(A) 2 (B) 58 (C) -6 (D) 6 (E) None of these

83. Which of the following functions is/are one-to-one?

(1) $f(x) = \dfrac{6}{x-13}$ (2) $g(x) = .05(x+3)$ (3) $h(x) = \sqrt{3x^2-40}$

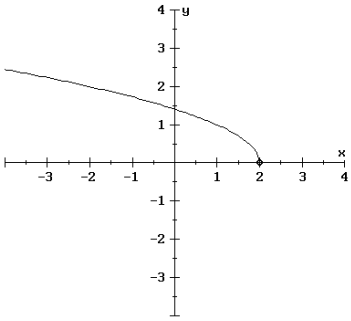
(A) 2 and 3 only (B) 1 and 2 only (C) 1, 2 and 3 only
(D) 1 and 3 only (E) None are one-to-one

84. If the graph of $S(x)$ is:

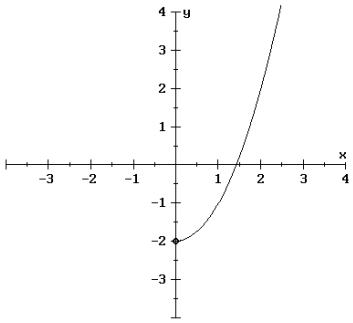


Then the graph of $S^{-1}(x)$ is:

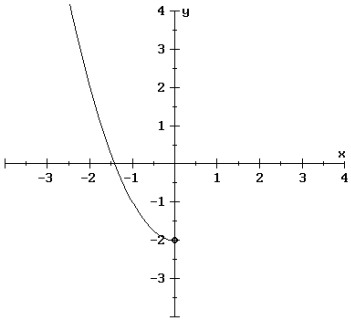
(A)



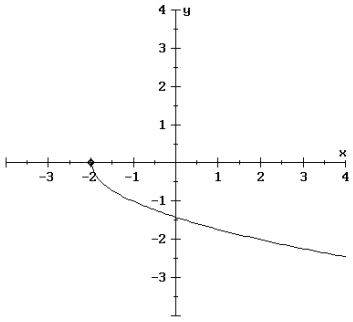
(B)



(C)



(D)



(E) None of these

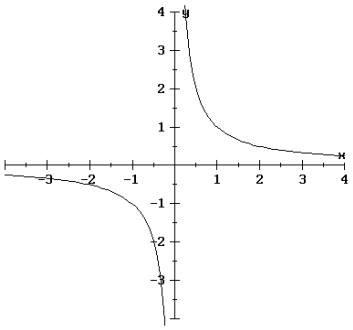
85. Find $R^{-1}(x)$ if $R(x) = \dfrac{C}{3x-1}$ (C is a real number)

(A) $R^{-1}(x) = \dfrac{C}{3}x + C$ (B) $R^{-1}(x) = \dfrac{3x-1}{C}$

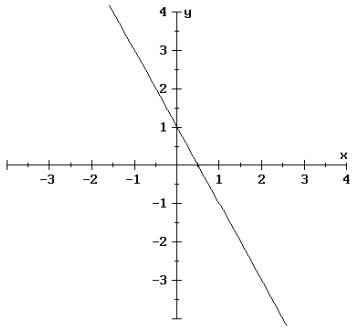
(C) $R^{-1}(x) = \dfrac{C+x}{3x}, x \neq 0$ (D) $R^{-1}(x) = \dfrac{C-3x}{x}, x \neq 0$

(E) None of these

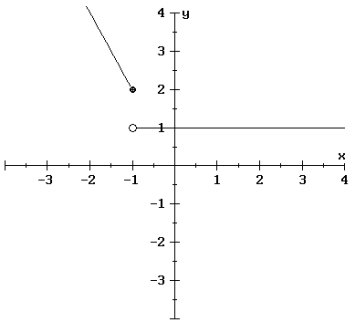
86. Of the functions graphed below, which have inverse functions?



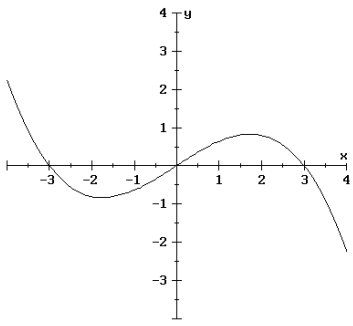
(1)



(2)



(3)



(4)

(A) 1, 2 and 3 only (B) 2 only (C) 1 and 2 only
(D) 1, 2 and 4 only (E) None of the answers A-D

87. Suppose $f(3) = 20$ means the volume of water in a container is 20 ounces when the depth of the water is 3 inches. What is the meaning of $f^{-1}(10) = 2$?

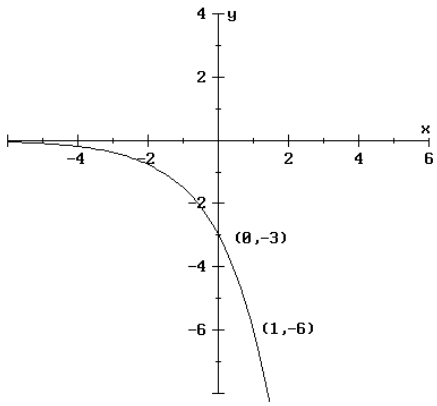
(A) The volume of water is 2 ounces when the depth of the water is 10 inches.
(B) The depth of the water is 2 inches when the volume of the water is 10 ounces.
(C) The depth of the water is 1/2 inch when the volume of the water is 10 ounces.
(D) Not enough information is given.

88. Which of the following is/are correct for the function $f(x) = a^{-x}(a > 1)$?

(1) $f(x)$ is decreasing (2) The domain is $(-\infty, 0]$
(3) The range is $(0, \infty)$ (4) The y-intercept is $(0, 1)$

(A) 1 and 4 only (B) 3 and 4 only (C) 1 only
(D) 1, 3 and 4 only (E) 2 only

89. The graph below represents $y = C(a)^x$. Find the values of C and a.



(A) $C = -3, a = 2$ (B) $C = -3, a = 3$ (C) $C = 2, a = -2$
(D) $C = 2, a = -3$ (E) $C = 1/3, a = -2$

90. Find the domain and range of the function $H(x) = 3^{x+2} - 4$.

(A) Domain: $(-2, \infty)$ Range: $(-4, \infty)$
(B) Domain: $(-\infty, \infty)$ Range: $(-4, \infty)$
(C) Domain: $(-\infty, \infty)$ Range: $(0, \infty)$
(D) Domain: $(2, \infty)$ Range: $(4, \infty)$
(E) Domain: $(-2, \infty)$ Range: $(4, \infty)$

91. Which of the following is true about the graph of $y = 5000e^{-.0002t} - 9000$?

(A) $y \longrightarrow \infty$ as $t \longrightarrow -\infty$ and $y \longrightarrow 9000$ as $t \longrightarrow \infty$
(B) $y \longrightarrow \infty$ as $t \longrightarrow -\infty$ and $y \longrightarrow -9000$ as $t \longrightarrow \infty$
(C) $y \longrightarrow 5000$ as $t \longrightarrow -\infty$ and $y \longrightarrow -9000$ as $t \longrightarrow \infty$
(D) $y \longrightarrow 0$ as $t \longrightarrow -\infty$ and $y \longrightarrow \infty$ as $t \longrightarrow \infty$
(E) $y \longrightarrow 5000$ as $t \longrightarrow -\infty$ and $y \longrightarrow 0$ as $t \longrightarrow \infty$

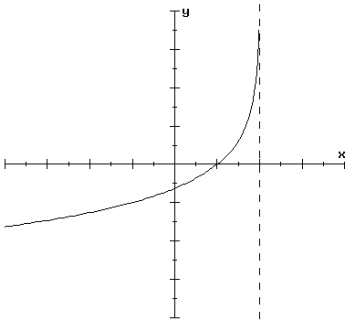
92. Which of the following is/are correct? $[a > 0, a \neq 1]$

(1) $\log_a 1 = 0$ (2) $\log_a 0 = 0$ (3) $\ln 1 = e$ (4) $\ln e^a = a$

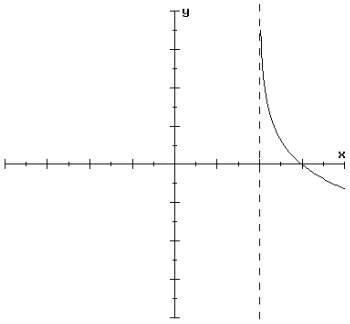
(A) None of them (B) 4 only (C) 1 and 4 only
(D) 3 only (E) 2 and 4 only

93. Which of the following most resembles the graph of $y = -\log_3(x - a)$, where a is a positive constant?

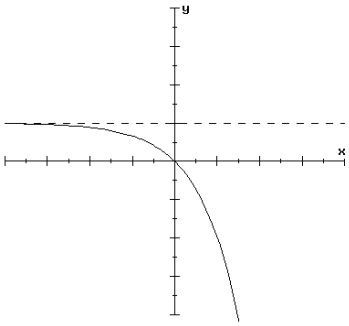
(A)



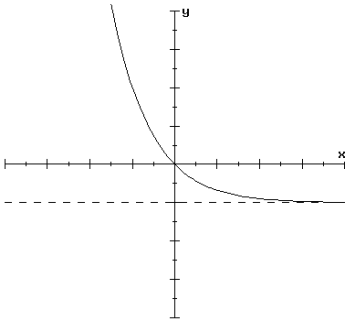
(B)



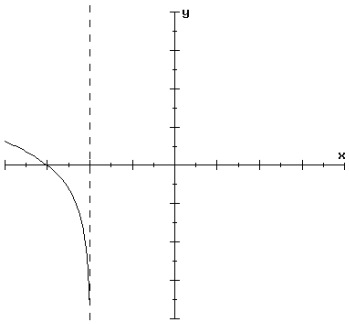
(C)



(D)

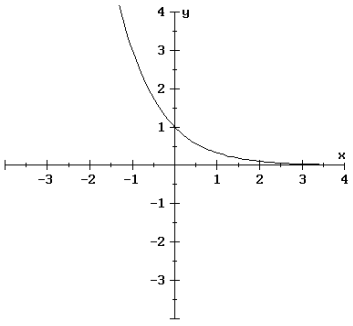


(E)

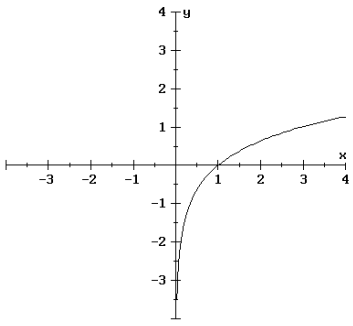


94. Which of the following is a graph of the INVERSE of $y = \log_3 x$?

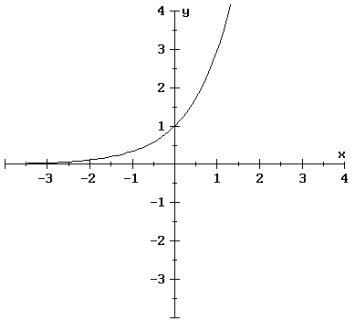
(A)



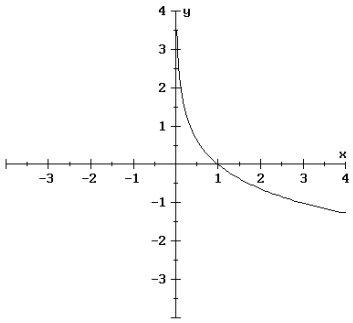
(B)



(C)



(D)



(E) None of these

95. What is the domain of $y = 10 - \log_7(5n - 230)$?

(A) $(-\infty, 230]$ (B) $[46, \infty)$ (C) $(46, \infty)$
(D) $(230, \infty)$ (E) $(-\infty, 48)$

96. Find the x-intercept of the graph of $M(x) = \log_6(2x + 3)$.

(A) $(1.5, 0)$ (B) $(0, 0)$ (C) $(-3, 0)$
(D) $(.5, 0)$ (E) $(-1, 0)$

97. For the natural log function $y = \ln(x)$, which of the following is/are correct?

(1) The graph is increasing (2) The x-intercept is $(1, 0)$
(3) The graph is continuous (4) The y-intercept is $(0, e)$

(A) All of them (B) 1, 2 and 3 only (C) 2, 3 and 4 only
(D) 2 and 3 only (E) 1 and 2 only

98. For the function $R(x) = \log_2 x$, which of the following is/are correct?

(1) The domain is $[0, \infty)$ (2) The range is $(-\infty, \infty)$ (3) $R(x)$ is one-to-one

(A) 2 and 3 only (B) 1 and 2 only (C) 3 only
(D) 1 and 3 only (E) All of them

99. Rewrite $23^b = a$ in logarithmic form.

(A) $\log_a b = 23$ (B) $\log_{23} a = b$ (C) $\log_{23} b = a$
(D) $\log_b 23 = a$ (E) None of these

100. Express as a sum or difference of the natural logarithms of x, y, and z: $\ln \sqrt{\dfrac{x^3 y}{z^4}}$

(A) $\dfrac{(3 \ln x)(\ln y)}{4 \ln z}$ (B) $\dfrac{3}{2} \ln x + \ln y - 4 \ln z$
(C) $\dfrac{3}{2} \ln x + \dfrac{1}{2} \ln y - 2 \ln z$ (D) $6 \ln x + 2 \ln y - 4 \ln z$
(E) None of these

101. Express as a single logarithm and simplify if possible:

$$\frac{1}{3}\log_a x + 4\log_a y - 2\log_a z$$

(A) $\log_a[x^{1/3} + y^4 - z^2]$ (B) $\log_a\left[\frac{1}{3}x + 4y - 2z\right]$ (C) $\frac{7}{3}\log_a\left[\frac{xy}{z}\right]$
(D) $\log_a\left[\frac{x^{1/3}y^4}{z^2}\right]$ (E) None of these

102. Find the exact value of $\ln\left[\sqrt[4]{e^5}\right]$.

(A) .8 (B) $\frac{5}{4}$ (C) e (D) $.8e$ (E) None of these

103. If M and N are positive, which of the following is/are correct?

(1) $\ln(MN) = \ln M + \ln N$ (2) $\ln(M+N) = \ln M + \ln N$

(3) $\frac{\ln M}{\ln N} = \ln M - \ln N$

(A) All of them (B) None of them (C) 1 only
(D) 1 and 3 only (E) 2 and 3 only

104. Solve for the variable in each equation: $\log_5 x = 2$ and $\log_2 32 = w$. The two solutions are:

(A) $x = \sqrt{5}$ and $w = \sqrt{32}$ (B) $x = 25$ and $w = 5$ (C) $x = 32$ and $w = 16$
(D) $x = 25$ and $w = 16$ (E) None of these

105. Solve for x : $3^{2x} = 27^{2x-1}$

The answer is a number:

(A) between .5 and 1 (B) between 0 and .5 (C) between $-.5$ and 0
(D) between 1 and 2 (E) None of these

106. Use natural logarithms to solve for x : $3 + 6e^{2x} = 5$

(A) $x = \frac{1}{2}\ln 3$ (B) $x = 2\ln 3$ (C) $x = \frac{1}{2}\ln\left(\frac{1}{3}\right)$
(D) $x = \ln 3 - 2$ (E) None of these

107. Solve for x : $3^x = 5^{x-1}$

The solution is a number:

(A) between 2 and 4 (B) between -5 and -3 (C) between -1 and 0
(D) between -3 and -1 (E) None of these

108. Solve for x: $\log_2(-4 - x) + \log_2(3 - x) = 3$

The solution is a number:

(A) between -9 and -6 (B) between -6 and -4
(C) between -4 and -1 (D) between -1 and 2
(E) between 2 and 5

109. Solve for x : $\ln(2x - 1) = 2$

(A) $x = \frac{e^2}{2}$ (B) $x = \frac{e^2 + 1}{2}$ (C) $x = e^2 + \frac{1}{2}$ (D) $x = \frac{e^4}{2}$
(E) None of these

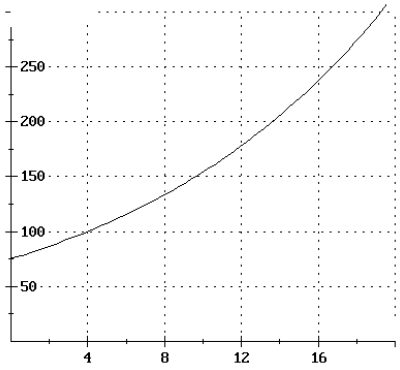
110. Solve for x : $\log_3 x - \log_3(x - 1) = 2$

(A) $\frac{1}{8}$ (B) $\frac{9}{8}$ (C) $\frac{10}{9}$ (D) $\frac{100}{99}$ (E) None of these

111. Find the x-intercept of the graph of $y = \ln(x - a) + 2$

(A) $(\ln(-a) + 2, 0)$ (B) $(-2 + \ln a, 0)$ (C) $(e^{-2+\ln(a)}, 0)$
(D) $(e^{-2} + a, 0)$ (E) None of these

It has been rumored that college costs have been growing exponentially. Suppose the cost of four years of college (in thousands of dollars) can be expressed as $y = Ce^{kt}$ where $t = 0$ corresponds to 1992, $t = 4$ corresponds to 1996, and so on. The graph of this function is shown below. Use this graph to answer the next TWO questions.



112. Use the graph (above) to find the value of C.

(A) 1992 (B) 75 (C) 0 (D) 100 (E) 150

113. What is the approximate value of k?

(A) .288 (B) .333 (C) .072 (D) .066 (E) .024

114. The number of California gray whales is growing according to the formula $A = Pe^{.015t}$, where t is measured in years. How long will it take the number of whales to double?

(A) less than 40 years (B) between 40 and 50 years
(C) between 50 and 60 years (D) between 60 and 70 years
(E) more than 70 years

115. In 1980, the population of the United States was approximately 226.5 million people. In 1990, the population had grown to approximately 246.7 million. Assuming an exponential growth model $A = Pe^{rt}$, what is the projected population of the U.S. in the year 2000?

(A) 266.9 million (B) 310 million (C) 268.7 million
(D) 212.6 million (E) None of these

116. Mr. Smart decided to invest \$20,000 in a savings account in which interest is compounded monthly at a fixed rate. What was the interest rate if he had \$36,500 in the account at the end of 10 years?

The interest rate is:

(A) less than 6.1% (B) between 6.1% and 6.3%
(C) between 6.3% and 6.5% (D) between 6.5% and 6.7%
(E) greater than 6.7%

117. How much MORE money will you earn in an account that compounds interest continually than in an account that compounds interest quarterly if you invest \$3000 for 7 years at an interest rate of 11%?

(A) \$67.02 (B) \$59.37 (C) 101.16 (D) \$32.52 (E) None of these

118. The release of fluorocarbons used in household sprays destroys the ozone layer in the upper atmosphere. Suppose the amount of ozone is given by $P = Ce^{-.0025t}$ where t is measured in years. How long will it take for 70% of the ozone to disappear? (Round to the nearest yr.)

(A) About 143 yrs. (B) About 1699 yrs. (C) About 1360 yrs.
(D) About 482 yrs. (E) None of these

119. A computer virus has been introduced into a system of computers. The data below was collected over the next 5 months:

Number of computers infected	Months since virus was introduced
24	1
96	2
384	3
1536	4
6144	5

Does the data represent a function which is linear or nonlinear? Which of the following equations best describes the number of computers infected, y, as a function of the time, t (measured in months)?

(A) nonlinear; $y = 6(4^t)$ (B) nonlinear; $y = 24(4^t)$ (C) linear; $y = 24 + 72t$
(D) linear; $y = 24 + t$ (E) None of these

120. Based on the pattern of the data from the question above, find the y-intercept of this function, and describe its practical significance.

(A) (0,6); there were no computers infected at time $t = 6$

(B) (0,6); 6 computers were initally infected

(C) (0,4); there were no computers infected at time $t = 6$

(D) (0,4); 4 computers were initially infected

121. If $\sum_{k=1}^{25} a_k = 40$ and $\sum_{k=1}^{25} b_k = 125$, find $\sum_{k=1}^{25} (3a_k - b_k + 2)$.

(A) -35 (B) 45 (C) -83 (D) 3 (E) None of these

122. Which of the following statements is/are TRUE?

(1) $\sum_{k=1}^{50}(15k^2+20k) = 5\sum_{k=1}^{50}(3k^2+4k)$

(2) $\sum_{i=5}^{40}(7) = 252$

(3) $\sum_{k=1}^{50}(5k^2-2) - \sum_{k=1}^{50}(k^2-2k) = \sum_{k=1}^{50}(4k^2+2k-2)$

(A) 1 only (B) All of them (C) 1 and 3 only
(C) 1 and 2 only (D) 3 only

123. Given that $\sum_{k=1}^{12}(2k^2+ck) = 1456$, determine the value of c.

(A) 4 (B) 0.5 (C) 1.5 (D) 3 (E) 2

124. Using the formula $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, find $\sum_{k=1}^{20}(2k^2+1)$.

(A) 2870 (B) 2890 (C) 4250 (D) 5780 (E) 5760

125. Evaluate the expression: $\sum_{m=0}^{4}(20-m!)$

(A) 66 (B) -16 (C) -4 (D) 16 (E) 90

126. Simplify: $\dfrac{(n+1)!}{2 \cdot n!}$

(A) $\dfrac{n+1}{2}$ (B) $\dfrac{1}{2n(n-1)}$ (C) $\dfrac{(n+1)(n-1)}{2}$

(D) $\dfrac{1}{n}$ (E) None of these

127. If the nth term of a sequence is given by $a_n = \dfrac{(-1)^{n-1}2^n}{(n+1)!}$,what is the fifth term?

(A) $-\dfrac{1}{24}$ (B) $\dfrac{2}{45}$ (C) $\dfrac{5}{3}$ (D) $-\dfrac{17}{121}$ (E) None of these

128. Write an expression for the nth term of the sequence: $4, 10, 18, 28, 40, ...$

(A) $a_n = 2n(n+1)$ (B) $a_n = n(n+3)$ (C) $a_n = n(n-3)$
(D) $a_n = n(n+1)$ (E) None of these

129. Find the $5th$ term of a sequence if $a_1 = 3$ and $a_n = 2a_{n-1} + 1$

(A) 63 (B) 11 (C) 42 (D) 54 (E) None of these

130. Write the terms of the sum: $\displaystyle\sum_{k=2}^{8} \frac{1}{k^2}$.

The third term is:

(A) $\dfrac{1}{9}$ (B)$\dfrac{1}{16}$ (C) $\dfrac{1}{25}$ (D) $\dfrac{1}{36}$ (E) None of these

131. Use sigma notation to write the sum: $\frac{2}{5}+\frac{4}{9}+\frac{6}{13}+\frac{8}{17}$

(A) $\sum_{k=1}^{4}\frac{k+1}{k+4}$ (B) $\sum_{k=1}^{4}\frac{3k-1}{3k+2}$ (C) $\sum_{k=1}^{4}\frac{2^k}{2k+3}$

(D) $\sum_{k=1}^{4}\frac{2k}{4k+1}$ (E) None of these

132. For an arithmetic sequence, if $d=3$ and $a_{39}=125$, find a_1.

(A) 9 (B) 12 (C) 11 (D) 8 (E) None of these

133. For an arithmetic sequence, if $a_1=\frac{3}{4}$ and $a_8=-\frac{11}{4}$, what is d?

(A) $-\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $-\frac{3}{8}$ (D) $-\frac{3}{2}$ (E) None of these

134. The $5th$ term of an arithmetic sequence is 16 and the 12th is 37. Find the common difference.

(A) 3 (B) $\frac{21}{6}$ (C) 7 (D) 21 (E) None of these

135. How many bricks will there be in a wall one brick in thickness if there are 39 bricks in the bottom row, 37 bricks in the second row, 35 bricks in the third row, and so forth, to the top row, which has one brick?

(A) 300 (B) 420 (C) 250 (D) 270 (E) None of these

136. Determine if the following sequence is arithmetic or geometric:

$$20, -10, 5, -\frac{5}{2}...$$

Find the $8th$ term of the sequence.

(A) $-\frac{5}{16}$ (B) $\frac{5}{32}$ (C) $\frac{33}{2}$ (D) $-\frac{5}{32}$ (E) None of these

137. Determine if the following sequence is arithmetic or geometric:

$$3, 7, 11, 15, \cdots$$

Find the 51st term of the sequence.

(A) 609 (B) 195 (C) 203 (D) 243 (E) None of these

138. Find the sum: $\sum_{n=1}^{9}(-2)^n$

(A) -1028 (B) -1266 (C) -342 (D) -482 (E) None of these

139. Find the 12th term of the geometric sequence given that the first term is -3 and the second term is 6.

(A) 96 (B) -2048 (C) -96 (D) 6144 (E) None of these

140. Determine if the following sequence is arithmetic or geometric or neither:

$$\frac{1}{3}, -\frac{2}{3}, \frac{4}{3}, -\frac{8}{3}, ...$$

Find the sum of the first 6 terms of the sequence.

(A) $\frac{11}{3}$ (B) -7 (C) $\frac{20}{3}$ (D) -28 (E) $-\frac{19}{3}$

141. Determine if the following sequence is arithmetic or geometric or neither:

$$2, 7, 12, 17, ...$$

Find the sum of the first 20 terms of the sequence.

(A) 2020 (B) 97 (C) 1980 (D) 99 (E) None of these

142. Parents decide to set up a college trust fund for their newborn child. The plan is to deposit \$100 a month for the next 18 years (i.e. 216 months) in a savings account that pays 6% annual interest compounded monthly. How much money will there be in the account at the end of 18 years? [Round to the nearest dollar]

$A = 100(1 + \frac{.06}{12})^1 + 100(1 + \frac{.06}{12})^2 + 100(1 + \frac{.06}{12})^3 + ... + 100(1 + \frac{.06}{12})^{216}$

(A) \$29,545 (B) \$1,888 (C) \$38,929 (D) \$52,953 (E) None of these

FORMULAS

$A_n = A_0\left(1+\frac{r}{m}\right)^{m\,n}$ $\qquad$ $A_n = A_0 e^{r\,n}$

$a_n = a_1 + (n-1)d$ $\qquad$ $a_n = a_1 r^{n-1}$

$f(n) = dn + c$ $\qquad$ $f(n) = cr^{n-1}$ where $c = a_1$

$S_n = n\left(\frac{a_1 + a_n}{2}\right)$ $\qquad$ $S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$S_n = a_1\left(\frac{1-r^n}{1-r}\right),\ r \neq 1$ $\qquad$ $S = \frac{a_1}{1-r},\ |r| < 1$

Summation Formulas:

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ $\qquad$ $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$