Practice Test #1
Solutions

Name:
Date: 13 Sept. 04

Math 110-054
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Instructions. Read all instructions carefully. The total time for this test is 25 minutes. ***An answer without supporting work is subject to no credit.*** This test will be worth 50 points. Read all the questions first so you can manage your time better. ***Watch for trick questions!***

(I) **Multiple Choice.** Circle your answer choices. If there are multiple correct answers, **circle** them all.

(1) (5 pts) Which of the following represent y as a function of x?

A. $2x^2 - y^2 = 1$

B. $\frac{x^2}{3} - 4 = y^3$

C. $2xy + y = 2$

D.

x	1.2	2.0	1.2	−0.5
y	−4	3	4	1

Solution: B, C

(2) (5 pts) Which of the following represent linear functions?

A. $f(x) = |2x + 4|$

B. $g(x) = \frac{2x}{4-5x}$

C.

x	1	2	3	4
y	2	4	8	16

D.

x	−2	2	4	6
y	9	17	21	25

E. $p(s) = 14 - 0.04s$

Solution: D, E

(II) **Written Answer.** Write all work, and complete sentences wherever possible. Remember, an answer without supporting work is subject to no credit.

(1) Determine the domains, y-intercepts, T-intercepts, and roots of the following functions.

(a) (7 pts) $f(T) = \sqrt{4 - T}$.
Solution: The domain of f is $4 - T \geq 0$, so $4 \geq T$. In interval notation, this is $(-\infty, 4)$.
The y-intercept is $f(0) = \sqrt{4-0} = \sqrt{4} = 2$.
The T-intercepts are $0 = \sqrt{4-T}$, so $0 = 4 - T$, so $4 = T$.
The roots are the same as the T-intercepts: $T = 4$.

(b) (7 pts) $h(T) = \frac{4}{\sqrt{4-T}}$.

Solution: The domain of h is $4 - T \geq 0$ and $\sqrt{4-T} \neq 0$, so $4 > T$ or $(-\infty, 4]$.
The y-intercept is $h(0) = \frac{4}{\sqrt{4-0}} = \frac{4}{2} = 2$.
The T-intercept is $0 = \frac{4}{\sqrt{4-T}}$, so $0 = 4$. This is a contradiction, so there is no T-intercept.
Since the roots of h are the T-intercepts, there are no roots.

(2) (7 pts) Solve the following equation for x. Write the solution set, and determine whether the equation is an identity, conditional, or contradiction.

(a) $x^2 + x - 1 = (2x+1)(x+2)$.

Solution: The right hand side becomes $2x^2+4x+x+2 = 2x^2+5x+2$. Simplifying the expression, we have

$$\begin{aligned} x^2 + x - 1 &= 2x^2 + 5x + 2 \\ 0 &= x^2 + 4x + 3 \end{aligned}$$

Using the quadratic formula, we get that

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} \\ &= \frac{-4 \pm \sqrt{4}}{2} \\ &= \frac{-4 \pm 2}{2} \\ &= -2 \pm 1 \\ &= -3 \text{ or } -1. \end{aligned}$$

So the solution set is $\{-3, -1\}$ (also $x = -3$ or -1), hence the equation is a conditional.

(3) (5 pts) A plumber charges \$560 for installing 40 feet of sewer pipe and \$780 for installing 60 feet. Find the average range of change of the cost as the length varies from 40 to 60 feet. Make sure to write your answer in a sentence, and to include units in your answer.

Solution: The average rate of change as the length varies from 40 to 60 feet is $\frac{\$780-\$560}{60 \text{ ft}-40 \text{ ft}} = \frac{\$220}{20 \text{ ft}} = \$11$ per foot.

(4) **Graph or sketch** each of the following functions, and determine its

- domain,
- range,
- y-intercepts,
- x-intercepts, and
- roots.

(a) (7 pts) $F(x) = |x - 1| + 2$.

Solution:

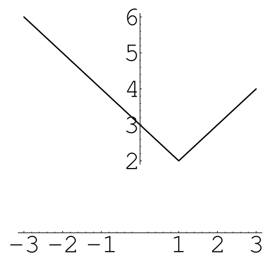

FIGURE 1. $F(x) = |x - 1| + 2$.

The domain of F is $\mathbb{R} = (-\infty, \infty)$, all real numbers, since absolute value makes sense for any real number.
The range of F is $[2, \infty)$, since the graph has a minimum at $(1, 2)$.
The y-intercept of F is $F(0) = |0 - 1| + 2 = |-1| + 2 = 1 + 2 = 3$.
It is clear from the graph that there are no x-intercepts. We can deduce this symbolically as well: We solve

$$(x - 1) + 2 = 0, \quad x > 1 \qquad \text{and} \qquad -(x - 1) + 2 = 0, \quad x \leq 1.$$

We do this and obtain the solutions $x = -1$ (when $x \geq 1$, a contradiction) and $x = 3$ (when $x \leq 1$, a contradiction).
The roots of a function are its x-intercepts, so F has no root.

(b) (7 pts) $F(x) = -\sqrt{1 - x^2}$.

Solution:

The domain of F is $[-1, 1]$: If $x > 1$ or $x < -1$, then $x^2 > 1$, so $0 > 1 - x^2$, and we can't take the square root of a negative number. If $-1 \leq x \leq 1$, then $x^2 \leq 1$, so $0 \leq 1 - x^2$. We can take the square root of a nonnegative number, so this is okay.

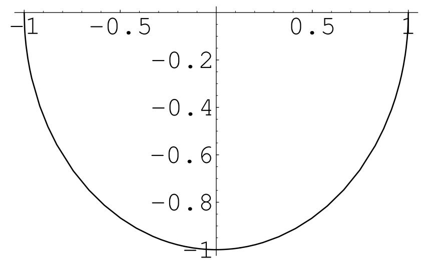

FIGURE 2. $F(x) = -\sqrt{1 - x^2}$.

Graphically, we see that the range of F is $[-1, 0]$.
The y-intercept of F is $F(0) = -\sqrt{1 - 0} = -1$.
The x-intercepts of F are when $0 = -\sqrt{1 - x^2}$. We square both sides to get $0 = 1 - x^2$. This simplifies to $x^2 = 1$, so $x = \pm 1$.
The roots are the same as the x-intercepts, so $x = \pm 1$.