Test #1

Name: **Math 110-054**
Date: 14 Sept. 04 **Tom LaGatta**

Instructions. Read all instructions carefully. The total time for this test is 50 minutes. ***An answer without supporting work is subject to no credit.*** This test will be worth 100 points. Read all the questions first so you can manage your time better. ***Watch for trick questions!***

(I) **Multiple Choice.** Circle your answer choices. If there are multiple correct answers, circle them all.

(1) (5 pts) Given $h(x) = \frac{4x+5}{3-x}$, what is the value of $h(2) - h(4)$?

A. 8 B. 34 C. 7 D. -34 E. None of these

Solution. B. $h(2) = \frac{8+5}{3-2} = 13$, and $h(4) = \frac{16+5}{3-4} = -21$, so $13 - (-21) = 34$.

(2) (5 pts) What is the domain of $h(x) = \frac{4x+5}{3-x}$?

A. $[3, \infty)$ B. $\{x \mid x \neq 3\}$ C. All real numbers ($\mathbb{R}$)

D. $[-\frac{4}{5}, \infty)$ E. $\{x \mid x \neq \frac{4}{5}\}$ F. None of these

Solution. B. The only place where h is undefined is $x = 3$, hence the domain is $\{x \mid x \neq 3\}$.

(3) For the linear function $f(x) = -3x + 7$, which of the following statements are true:

(3 pts) A. The function is increasing.
False. The function is decreasing.

(2 pts) B. For each unit increase in the input, the output decreases by 3 units.
True. Rise/run.

(3 pts) C. The slope of this function is negative.
True. The slope is -3.

(1 pt) D. The graph intersects the x-axis as the point $(0, 7)$.
False. The graph intersects the y-axis at the point, not the x-axis.

(1 pt) E. This function has a decreasing rate of change.
False. The graph has a constant rate of change: -3.

(II) **Written Answer.** Write all work, and complete sentences wherever possible. Remember, an answer without supporting work is subject to no credit.

(1) (10 pts) Write an equation for the line passing through the points $(-2, -9)$ and $(4, 12)$.
Solution. First, find the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - (-9)}{4 - (-2)} = \frac{21}{6} = \frac{7}{2} = 3.5.$$

The equation for the line is then

$$y - 12 = 3.5(x - 4) \quad \text{or} \quad y + 9 = 3.5(x + 2).$$

You could also expand it in slope-intercept form to get

$$y = 3.5x - 2.$$

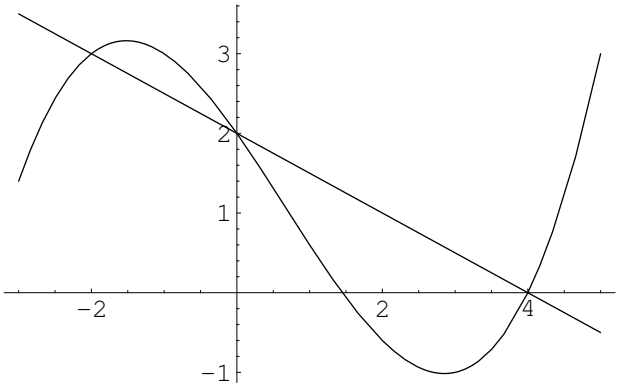
Any of these answers is acceptable.

(2) (15 pts) Write a formula for a linear function $f(x)$ that models the data in the following numerical representation exactly.

x	-2	0	4
y	3	2	0

Is $f(x)$ the only line that satisfies this data? Why or why not?
Is there some nonlinear function $g(x)$ that also satisfies the data?
Graph f and g on the same coordinate axes.

Solution. First check that the average rates of change are the same. For every run of 2, there is a rise of -1. So the data can be modelled by a line, with slope $m = -\frac{1}{2}$.
$f(x) = y = -\frac{1}{2}x + 2$ is the only linear function that models this data. Any other line must have the same slope, hence is parallel, and must hit the same points, hence must be the same line.
There are nonlinear functions g that hit these points too:



(3) (7 pts) Solve the following equation for x. Write the solution set, and determine whether the equation is an identity, conditional, or contradiction.

(a) $\sqrt{3}(x+1) = \sqrt{3}x + \frac{\sqrt{3}}{3}$.

Solution. Factor the left side to get

$$\sqrt{3}x + \sqrt{3} + \frac{\sqrt{3}}{3}.$$

Subtract $\sqrt{3}x$ from both sides to get

$$\sqrt{3} = \frac{\sqrt{3}}{3}.$$

This is a *contradiction*–if you take any nonzero number and divide it by 3, you will get a different number. So, there are no solutions to this equation.

(4) **Graph or sketch** each of the following functions, and determine its
- domain,
- range,
- y-intercepts,
- X-intercepts, and
- roots.

(a) (12 pts) $F(X) = X^3 - 2X^2 + X$.

Solution.

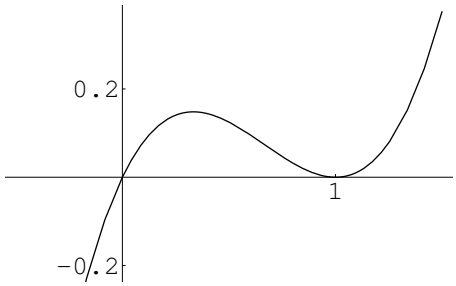


FIGURE 1. $F(X) = X^3 - 2X^2 + X$.

Factor $F(X)$ to get $X(X^2-2X+1) = X(X-1)(X-1) = X(X-1)^2$. (Once you factor out the first X, you can use the quadratic formula.)
The domain is all real numbers, since $F(X)$ is a *polynomial* (scary word). We see that the graph is defined everywhere.
The range is $\mathbb{R} = (-\infty, \infty) =$ all real numbers, since $F(X)$ has degree 3. (That comes from the 3 in the expression $X^3 - 2X + X$). We talked in class that the range of any **odd** degree *polynomial* (scary word) is all real numbers. You could also see this from the graph.
To get y-intercepts, we plug in $X = 0$. This is $F(0) = 0^3 - 2 \cdot 0 + 0 = 0$. So the y-intercept is 0 or $(0, 0)$ (either is acceptable).
To get X-intercepts, you factor the equation and set it equal to 0. So $F(X) = X(X-1)^2 = 0$. This means that either $X = 0$ or $X = 1$. You could also determine this from the graph, since the graph hits the x-axis at 0 and 1.
The roots are exactly the same as the X-intercepts: $x = 0$ or $x = 1$.

(b) (12 pts) $F(X) = mX + b$, with $m > 0$ and $b > 0$.

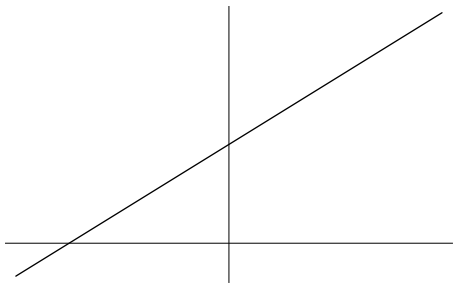


FIGURE 2. $F(X) = mX + b$.

F here is a diagonal line, so the domain and range are all real numbers $\mathbb{R}$. You can also tell this by the graph.
The y-intercept is b, since the equation is in slope-intercept form.
To get the x-intercepts and roots, set the equation equal to 0. So $0 = mX + b$, $mX = -b$, then $X = -\frac{b}{m}$. I believe this was a homework problem as well.

(5) (6 pts) Old Man Sinclair has a circular garden of candy canes and sunshine dust, but those neighborhood brats keep sneaking in after dark and draining him of his riches. To protect his treasure trove of sunshine, the crusty geriatric wants to build a fence constructed from pure agony. If the area of his garden is 100π m^2, what length of fence will Old Man Sinclair need to guard his personal heaven? (Hint: Draw a picture)

Solution. We know that the area of a circle is $A = \pi r^2$, and the circumference is $C = 2\pi r$. We know that the area of Sinclair's garden is 100π m^2, so $A = \pi(10 \text{ m})^2$. This means that the radius $r = 10$ m. Plugging into the circumference equation, $C = 2\pi r = 2\pi(10 \text{ m}) = 20\pi$ m. So, Old Man Sinclair needs to purchase 20 m ≈ 62.8 m of agony fence.

(3 pts) If the local hardware store charges \$9.55 $\approx$ \$$\frac{30}{\pi}$ per meter of agony fence, how much will Old Man Sinclair spend securing his fortune?

Solution. From the above, we know that Sinclair needs to purchase 20π m of agony fence. Since each meter of fence costs \$$\frac{30}{\pi}$, Sinclair will have to spend about

$$20\pi \text{ m} \cdot \frac{30}{\pi} \frac{\text{dollars}}{\text{m}} = \$600.$$

(6) Define the function

$$f(x) = \begin{cases} -2x + 1, & -3 \leq x < 0 \\ x - 1, & 0 \leq x \leq 3 \\ 5, & x > 3. \end{cases}$$

(a) (6 pts) Find $f(0.5)$ and $f(4)$.

Solution. $0 \leq 0.5 \leq 3$, so we use the second formula. Then

$$f(.5) = .5 - 1 = -.5 = -\frac{1}{2}.$$

$4 > 3$, so we use the third formula. Then

$$f(4) = 5.$$

(b) (5 pts) Draw the graph of f.

(No) Solution. I can't seem to draw this graph using a computer, so I'll draw it in class Thursday or can show you in office hours.

(c) (4 pts) What is the domain of f? The range?

Solution. Looking at the graph, you can see that it is defined for $-3 \leq x < \infty$. So the domain is $[-3, \infty)$. The range is $[-1, 5]$, since we get outputs as low as -1, as high as 5, and hit everything in between. (this comes from the graph).