Test #2
Form A
Solutions

Name: **Math 110-054**
Date: 12 Oct. 04 **Tom LaGatta**

Read all instructions carefully. Read all the questions first so you can manage your time best.
The total time for this test is 50 minutes. The test will be worth 100 points
You will turn in only the answer sheet, so make sure all your work and answers are on the answer sheet. Put your name at the top of every page.
An answer without supporting work is subject to no credit.
Put exact solutions wherever possible (e.g., if an answer is $\sqrt{2}$, write $\sqrt{2}$, not 1.4). *Circle your answers.*
Watch for trick questions!

Part A
Multiple Choice. (You do not have to show work).

1. (5 points) For the polynomial function $f(x) = -9x^6 - 6x^5 + 3x^4 - 1.5x^2 - 17$, which of the following are true?

(i) $f(x)$ is an odd degree polynomial.

(ii) $f(x)$ is an even degree polynomial.

(iii) $f(x)$ is neither an even function nor an odd function.

	Answer choices		
A. i.	B. ii.	C. iii.	D. i, ii, and iii
E. i and ii	F. i and iii	G. ii and iii	H. None of the above.

Solution

The solution is G. The degree of f is 6, so (i) is false and (ii) is true. f is neither even nor odd (check that $f(-1) \neq f(1)$ nor $-f(1)$), so (iii) is true.

2. (5 points) For the polynomial function $f(x) = -9x^6 - 6x^5 + 3x^4 - 1.5x^2 - 17$, which of the following are true?

(i) The leading coeffecient of f is positive.

(ii) The end behavior of this polynomial is $x \to \infty$, $y \to -\infty$, and $x \to -\infty$, $y \to \infty$.

(iii) The end behavior of this polynomial is $x \to \infty$, $y \to -\infty$, and $x \to -\infty$, $y \to -\infty$.

	Answer choices		
A. i.	B. ii.	C. iii.	D. i, ii, and iii
E. i and ii	F. i and iii	G. ii and iii	H. None of the above.

Solution

The leading coeffecient is negative, meaning (i) is false, so the polynomial opens down, so (ii) is false and (iii) is true.

3. (5 points) How many zeros could a fifth degree polynomial have?

Answer choices

A. 0, 1, 2, 3, 4, 5, 6.	B. $\mathbb{R}$.	C. 1, 2, 3, 4, 5.
D. 0, 1, 3, 4.	E. 0, 1, 2, 3, 4, 5.	F. 1, 2, 3, 4, 5, 6

Solution

C. Remember that an n degree polynomial has at most n zeros–this eliminates A and F. An odd degree polynomial always has at least 1 zero, eliminating D and E. B doesn't make sense in this context. The answer is C.

4. (5 points) Complete the following table to make the function an odd function:

x	-5	-3	-2	-1	0	1	2	3	5
y	-1	0		1			8	0	

Answer choices

A. 0, 1, 1, 8	B. -8, 0, -1, 1	C. 0, 1, -1, 8	D. -8, 0, 1, -1
E. 8, 0, 1, -1	F. 0, -1, -1, 8	G. 8, 0, -1, 1	H. 0, -1, 1, 8

Solution

B. For a function to be odd, it must satisfy $f(-x) = -f(x)$. So, $f(-2) = -f(2) = -8$, $f(1) = -f(-1) = -1$, and $f(5) = -f(-5) = 1$, and $f(0) = -f(0)$, so $2\,f(0) = 0$, hence $f(0) = 0$.

5. (5 points) Consider the quadratic $523.492(x - .2437)^2 - 2.139$. Which of the following are true?

(i) The graph is concave up (opens upward).

(ii) The graph is concave down (opens downward).

(iii) The x-coordinate of the vertex is -2.139.

Answer choices

A. i only	B. ii only	C. iii only	D. i, ii, and iii
E. i and ii only	F. i and iii only	G. ii and iii only	H. None of the above.

Solution

The leading coordinate of the quadratic is 523.492, which is positive, so the graph opens upward. So 1 is true and 2 is false. The vertex is $(.2437, -2.139)$, which has x-coordinate $.2437 \neq -2.139$. So 3 is false. The correct answer is A.

6. (5 points) You are given two lines, each with *different* positive slopes. How many points of intersection will occur?

Answer choices

A. 0 B. 2 C. Not enough information.
D. 1 E. None of the above.

Solution

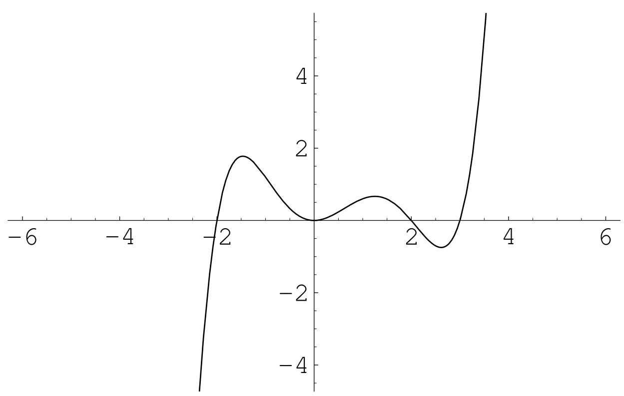
Algebraically: Let $f(x) = m_1x + b_1$, and $g(x) = m_2x + b_2$. Setting these equal, we have $m_1x + b_1 = m_2x + b_2$. Putting like terms on the same side, we have $(m_1 - m_2)x = b_2 - b_1$. Since $m_1 \neq m_2$, $m_1 - m_2 \neq 0$, so we can divide:

$$x = \frac{b_2 - b_1}{m_1 - m_2}.$$

Hence there is always exactly one point of intersection at $\frac{b_2 - b_1}{m_1 - m_2}$.

You can also show this graphically.

7. (5 points) How many zeros of multiplicity 2 does the following graph have?



Answer choices

A. 0 B. 1 C. 2
D. 3 E. 4 G. None of the above

Solution

The graph has 1 zero of multiplicity 2, and 3 zeros of multiplicity 1. The correct answer is B.

8. (5 points) How many turning points does the above graph have?

Answer choices

A. 0 B. 1 C. 2
D. 3 E. 4 G. None of the above

Solution

The graph has 4 turning points. The correct answer is E.

9. (5 points) If $(2, -5)$ is on the graph of $f(x)$, then what point must be on the graph of $y = f(2x) + 2$?

Answer choices

A. $(0, -10)$	B. $(1, -7)$	C. $(1, -3)$
D. $(4, -3)$	E. $(4, -7)$	F. None of the above.

Solution

y is a transformation of $f(x)$. In particular, it is a horizontal shrink of 2, and a vertical shift 2 up. So the point $(2, -5)$ goes to $(1, -3)$, answer choice C.

Part B

Written answer. (You have to show work).

1. (10 points) Find the roots of the cubic polynomial $f(y) = y^3 + y^2 - 3y - 2$.

Solution

Use your calculator or plug in some small values for y to get that

$$f(-2) = -8 + 4 + 6 - 2 = 0,$$

so $y + 2$ is a factor of f. Now use long division or synthetic division

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -3 & -2 \\ & & -2 & 2 & 2 \\ \hline & 1 & -1 & -1 & 0 \end{array}$$

we get that $f(y) = (y+2)(y^2 - y - 1)$. We use the quadratic formula on the second part to get

$$y = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}.$$

The roots of f are $y = -2$, $\frac{1+\sqrt{5}}{2}$, and $\frac{1-\sqrt{5}}{2}$.

2. (8 points) The quadratic $y = x^2 - 6x + 2$ is the result of which transformation(s) of $y = x^2$? (Hint: Put y into vertex form.)

Solution

Using the completing-the-square formulas, we know that

$$y = a(x - h)^2 + k,$$

where $h = \frac{-b}{2a} = \frac{6}{2} = 3$ and $k = c - ah^2 = 2 - 1 \cdot 3^2 = 2 - 9 = -7$. Hence $y = (x-3)^2 - 7$.

This is is a translation 3 right and 7 down of $y = x^2$.

3. (10 points) Old Man Sinclair is teaching his adoptive son to play ball. According to Dr. Spock, the most effective way to do this is to use a cannon. The geezer bought his boy a \$427.28 top-of-the-line 128 ft/sec cannon from the local hardware store, and is all set to show him how it's done. The birds know the deal, though, so they're flying just out of cannon range. If Old Man and Young Boy Sinclair set up the cannon on the roof of Chez Sinclair, which is fifteen feet above the ground, then how low can the birds go? (Hint: $h(t) = -32t^2 + v_0 t + h_0$. What is the initial velocity/speed? Initial height?)

Solution

Set the initial velocity/speed as $v_0 = 128\,\text{ft/sec}$, and the initial height as $h_0 = 15\,\text{ft}$. Then $h(t) = -32t^2 + 128t + 15$. To find the maximum height, find the maximum of this quadratic, that is, the vertex. Using the formulas, we have $h = -b/2a = 128/64 = 2$ and $k = c - ah^2 = 15 + 32(4) = 15 + 128 = 143$. Hence the maximum is at $(2\,\text{sec}, 143\,\text{ft})$. The bird's can fly just above 143 ft and stay safe.

4. (9 points) For the quadratic equation $3x^2 - 2qx + 4 = 0$, *where q is a constant*, what value(s) for q will make this equation have exactly one real solution?

Solution

Use either the discriminant or quadratic formula. Using the formula, we have

$$x = \frac{2q \pm \sqrt{4q^2 - 4(3)(4)}}{6} = \frac{2q \pm 2\sqrt{q^2 - 12}}{6} = \frac{q \pm \sqrt{q^2 - 12}}{3}.$$

For there to be only one solution, the + and − parts must be the same, which only happens when $\sqrt{q^2 - 12} = 0$. We solve this equation, getting $q^2 = 12$, so

$$q = \pm\sqrt{12} = \pm 2\sqrt{3}.$$

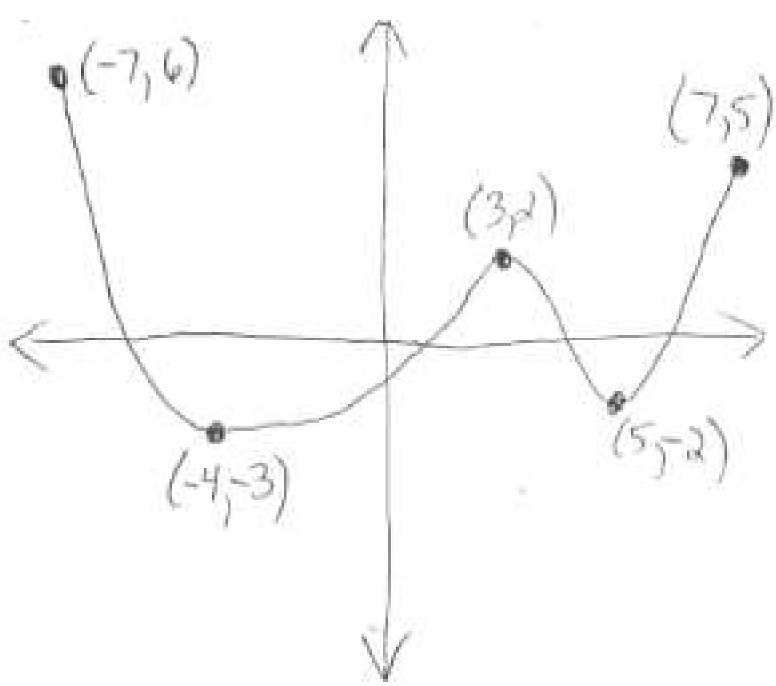
5. (8 points) Determine a fourth degree polynomial with zeros at 3, −1, $\sqrt{10}$, and $-\sqrt{10}$.

Solution

Let

$$f(x) = (x - 3)(x + 1)(x - \sqrt{10})(x + \sqrt{10}).$$

6. (10 points) Given the graph of $y = f(x)$ below, find the following:



(a) The turning points.

(b) The absolute minimum.

(c) All local extrema.

(d) The domain.

(e) The intervals where f is decreasing.

Solution

(a) The graph has turning points at $(-4,-3)$, $(3,2)$, and $(5,-2)$.

(b) The graph has its absolute minimum at $(-4,-3)$. The minimum value is -3.

(c) The graph has local extrema at the hills and troughs. Here the local maxima are at $(3,2)$, and the local minima are at $(-4,-3)$ and $(5,-2)$. Note that the endpoints are not local extrema.

(d) The domain of f is $-7 \leq x \leq 7$.

(e) f is decreasing from -7 to -4, and again from 3 to 5.