Test 3
Solutions
Form A

Name: **Math 110-054**
Date: 5 Nov. 04 **Tom LaGatta**

Read all instructions carefully. Read all the questions first so you can manage your time best.
The total time for this test is 50 minutes. The test will be worth 100 points.
You will turn in only the answer sheet, so make sure all your answers are on the answer sheet. Put your name at the top of every page.
Watch for trick questions!

Formulas: $A_n = A_0\left(1+\frac{r}{m}\right)^{mn}$ and $A_n = A_0 e^{rn}$.

Part A
Multiple Choice. You do not have to show work. Partial credit will not be given.

1. (5 points) Given $f(x) = 6x - 2$ and $g(x) = |x-2|$, find $\left(\frac{f}{g}\right)(-4)$.

Answer choices

A. 13 B. $\frac{13}{3}$
C. -13 D. $-\frac{13}{3}$

Solution

Answer: $-\frac{13}{3}$. Plug in $\left(\frac{f}{g}\right) = \frac{6x-2}{|x-2|}$. At -4, this is $\frac{-24-2}{|-4-2|} = \frac{-26}{|-6|} = \frac{-13}{3}$.

2. (5 points) Which of the following is/are correct? ($a > 0$ and $a \neq 1$)

(1) $\log_a 1 = 0$ (2) $\log_a 0 = 0$ (3) $\ln 1 = e$ (4) $\ln e^a = a$

Answer choices

A. (4) only B. (1) and (4) only C. (1), (2), and (3) only
D. (3) only E. (2) and (4) only F. None of these.

Solution

Answer: (1) and (4) only. a to what power is 1? 0, so (1) is true. a to what power is 0? None, so (2) is false. e to what power is 1? 0, so (3) is false. e to what power is e^a? a, so (4) is true.

3. (5 points) Find the x-intercept of the graph of $M(x) = \log_3(2x-6)$.

Answer choices

A. $(1, 0)$	B. $(3.5, 0)$	C. $(-1, 0)$
D. $(3, 0)$	E. $(1.5, 0)$	F. None of these.

Solution

Answer: $(3.5, 0)$. M has an x-intercept when $2x - 6 = 1$, since $3^0 = 1$.

4. (5 points) Given $G(x) = \frac{x^3 - x^2 - x}{x^2 - 1}$, find the SUM of the zeros of G.

Answer choices

A. $\frac{1+\sqrt{5}}{2}$	B. -1	C. 0
D. $\frac{1-\sqrt{5}}{2}$	E. 1	F. None of these.

Solution

Answer: 1. Factor

$$M(x) = \frac{x(x^2 - x - 1)}{x^2 - 1} = \frac{x(x - \frac{1-\sqrt{5}}{2})(x - \frac{1+\sqrt{5}}{2})}{(x-1)(x+1)}.$$

M has a zero when top $= 0$ and bottom $\neq 0$, so the zeros are $x = 0$ and $\frac{1 \pm \sqrt{5}}{2}$. The sum is then $\frac{1+\sqrt{5}}{2} + \frac{1-\sqrt{5}}{2} = 1$.

5. (5 points) Given G as above, find the holes of G, if any.

Answer choices

A. $(1, -1)$	B. $(1, \frac{\sqrt{5}-1}{2})$	C. $(0, 0)$
D. $(-1, 1)$	E. $(-1, \frac{\sqrt{5}-1}{2})$	F. None of these.

Solution

Answer: None of these. There is possibly a zero when the top and bottom are both equal to 0, but this never happens for G.

6. (5 points) Given G as above, find the horizontal or slant asymptote of G, if it exists.

Answer choices

A. $y = -1$	B. $y = -2x + 1$	C. $y = 3/2$
D. $y = x - 1$	E. $y = 0$	F. None of these.

Solution

Answer: $y = x - 1$. deg top = deg bottom + 1, so G has a slant asymptote. Long division results in $G = x - 1 + \frac{-1}{x^2-1}$, so the slant asymptote is $y = x - 1$.

7. (5 points) What is the domain of $\log_3(x)$?

Answer choices

A. $\{x \mid x > 0\}$ B. $\{x \mid x > 1\}$ C. $\mathbb{R}$

D. $\{x \mid x \geq 0\}$ E. $\{x \mid x \geq 1\}$ F. None of these.

Solution

Answer: $\{x \mid x > 0\}$. The domain of any logarithm is $x > 0$.

8. (5 points) Find the inverse function of $F(t) = At^2 + B$, if it exists. A and B are constants.

Answer choices

A. $\sqrt{\frac{t-A}{B}}$ B. $\sqrt{\frac{t-B}{A}}$

C. $\sqrt{t - B - A}$ D. None of these.

Solution

Answer: None of these. F is not 1-1, so no inverse exists.

9. (5 points) Which of the following is/are correct for the function $f(x) = a^{-x}$, for $a > 1$?

(1) $f(x)$ is decreasing. (2) The domain is $(-\infty, 0]$.

(3) The range is $(0, \infty)$. (4) The y-intercept is $(0, 1)$.

Answer choices

A. (1) and (4) only B. (3) and (4) only C. (1), (3), and (4) only

D. (1) only E. (2) only F. None of these.

Solution

Answer: (1), (3), and (4) only. $f(0) = a^0 = 1$, so (4) is true. The domain of f is all real numbers, so (2) is false. (1) and (3) we see are true from the graph of f.

10. (5 points) Which of the following is NOT correct?

Answer choices

A. $\log_4 x - \log_4 z + \log_4 y = \log_4 \left(\frac{xy}{z}\right)$

B. $\log_7(16x^2) = 2\log_7(4x)$

C. $\ln 1 = 0$

D. None of these.

Solution

Answer: None of these. All of the above are true.

11. (5 points) Suppose you wish to invest \$3,300 at 2.7% interest for 32 years. What is the difference in

the value of your investment if you invest it compounded continuously as opposed to compounded every six months?

Answer choices

A. \$0	B. \$45.12	C. \$15,644.20
D. \$7.60	E. \$15.16	F. None of these.

Solution

Answer: \$45.12. Use the formulas $A_n = A_0 e^{rn}$ and $A_n = A_0\left(1+\frac{r}{m}\right)^{mn}$, then take their difference.

12. (5 points) Which of the following functions have/has NO vertical asymptote?

$$f(x) = \frac{3x^3}{2x^2} \qquad g(x) = \frac{3x+1}{x^2+1} \qquad h(x) = \frac{2}{x^2-5}$$

Answer choices

A. f only	B. h only	C. f, g, and h
D. g and h	E. f and g	F. None of these.

Solution

Answer: f and g only. f has only a hole at $(0,0)$, the denominator of g has no roots, and h has vertical asymptotes at $x = \pm\sqrt{5}$.

13. (5 points) Which of the following statement(s) is/are true about the function

$$f(x) = \frac{-6x+1+9x^2}{x-3}?$$

(1) $y \to -6$ as $x \to \infty$.
(2) $f(x)$ has exactly one x-intercept.
(3) $f(x)$ has a slant asymptote.

Answer choices

A. (1) and (2) only	B. (2) and (3) only	C. (1) only
D. (2) only	E. (3) only	F. None of these.

Solution

Answer: (2) and (3) only. deg top = deg bottom + 1, meaning f has a slant asymptote, so (3) is true and (1) is false. We factor the top to get $(-3x+1)^2$, hence f has only one x-intercept: $x = \frac{1}{3}$.

14. (5 points) If $f(x)$ is a one-to-one function, and $f(-3) = 2$, then which of the following CANNOT be true?

Answer choices

A. $f(2) = -3$ B. $f^{-1}(2) = -3$ C. $f^{-1}(0) = -2$

D. $f(3) = 1$ E. $f(3) = 2$ F. None of these.

Solution

Answer: $f(3) = 2$. Since f is one-to-one, no output can be repeated.

15. (5 points) If Old Man Sinclair has a half-life of 7 years, then how long will it take for Old Man Sinclair to decay to 8%?

Answer choices

A. 26 years B. 21 years

C. 2 years D. None of these

Solution

Answer: 26 years. Use the half-life formula $f(t) = \left(\frac{1}{2}\right)^{t/k}$. We want to find t such that

$$.08 = \left(\frac{1}{2}\right)^{t/7}.$$

Taking the log-base-2 of both sides, we have

$$\log_2(.08) = \log_2\left(\frac{1}{2}\right)^{t/7} = \frac{t}{7}\log_2\left(\frac{1}{2}\right) = \frac{t}{7}\log_2\left(2^{-1}\right) = -\frac{t}{7}\log_2(2) = -\frac{t}{7}.$$

Solving for t, we get $t = -7\log_2(.08) \approx 25.5 \approx 26$.

Part B

Written Answer. You must show work. **Box your final answer**. Exact answers only–no decimal approximations.

1. (15 points) Solve for the variable and simplify as much as possible. You must use properties of exponentials and logarithms—do not just enter the expression in your calculator.

(a) $y = \log_3 \frac{1}{27}$

(b) $3 \cdot 2^x = 192$

(c) $\log_4 n = \frac{1}{4}$

Solution

2. (10 points) For this problem you *must* write in complete sentences.

(a) Can a one-to-one function have more than one x-intercept? If so, give an example. If not, explain.

(b) Can it have more than one y-intercept? If so, give an example. If not, explain.

Solution

Answer: No function can have more than one y-intercept, otherwise it would fail the vertical line test. A one-to-one function cannot have more than one x-intercept, otherwise it would fail the horizontal line test.