

HOMEWORK 11
DUE WEDNESDAY, 16 APRIL 2008

MATH 215 - LINEAR ALGEBRA - TOM LAGATTA

- Read pages
 - 253-273 (Sections 4.1 and 4.2 up to Cramer's Rule)
 - 279-280 (Brief History of Determinants)
 - 45, 283-285 (Cross Product and Geometric Applications of Determinants)

- (Problem A) Remember problem 57 from Section 3.5: If A is $m \times n$ and B is $n \times r$, then

$$\text{rank}(AB) \leq \text{rank}(B).$$

Here is a different approach this proof, which uses geometry instead of algebra. We can think of A as a function (or linear transformation):

$$\begin{aligned} A &: \mathbb{R}^n \rightarrow \mathbb{R}^m \\ \vec{v} &\mapsto A\vec{v} \end{aligned}$$

which sends a vector $\vec{v} \in \mathbb{R}^n$ to the matrix product $A\vec{v} \in \mathbb{R}^m$. Similarly for B :

$$\begin{aligned} B &: \mathbb{R}^r \rightarrow \mathbb{R}^n \\ \vec{w} &\mapsto B\vec{w} \end{aligned}$$

- (A1) In terms of subspaces we have talked about, what are the images of A and B ?
- (A2) These are subspaces of which larger spaces?

Now, think about the composition of these two functions: We take a vector \vec{v} , send it through the function B to get $B\vec{v}$, then send *that* through A to get $A(B\vec{v}) = (AB)\vec{v}$.

- (A3) Argue that $\dim(\text{image}(AB))$ is *at most* as large as $\dim(\text{image}(B))$.
 - (A4) Explain why this means that $\text{rank}(AB) \leq \text{rank}(A)$.
- **Section 3.6:** 3, 4, 5, 7, 9, 11, 12, 13, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25

Comments:

- For problems 16 and 21, draw a picture and use some trigonometry.
- For problems 22 and 23, use the Projection Formula on page 24. Plug in some example points, then plug in a generic point (x, y) / vector $[x, y]$.

(continued on next page)

- **Section 4.1:** 3, 5, 7, 11, 19, 20, 21, 22, 23, 24, 25, 27, 28, 35, 36, 37

Comments:

- I will definitely be grading at least one of 35, 36, or 37, so make sure to do them.
- Definition: The *trace* of a matrix is the sum of its diagonal entries. For the 2×2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

$$\text{tr}(A) = a + d.$$

- In light of Problem 35, think about why I call eigenvalues SUPER IMPORTANT.
- (Problem B) For the matrices you found in **Section 3.6** Problems 15, 16, 17, 18,
 - (b) find all real eigenvalues and eigenvectors (or demonstrate why there are none),
 - (c) explain what these represent geometrically.