

HOMEWORK 11
DUE WEDNESDAY, 16 APRIL 2008

MATH 215 - LINEAR ALGEBRA - TOM LAGATTA

- Read pages
– 289-295 (Sections 4.3 and 4.4)
- **Section 4.3:** 1, 3, 4, 7, 10, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24
- **Exam Review.**
- The third exam will be held **Wednesday, April 30**. It will chiefly cover Sections 3.3, 3.5, 3.6, 4.1, 4.2, and 4.3.
- The final exam will be held **Friday, May 16** from 5-7pm in PSYCH 307. It will be comprehensive, and will also cover Section 4.4.
- I don't want you to fall out of practice, so row reduce by hand the matrix

$$A = \begin{bmatrix} 2 & 4 & 1 & 1 & -1 \\ 1 & 2 & -2 & 3 & 0 \\ -2 & -4 & 1 & -3 & -1 \\ 1 & 2 & 0 & 1 & 2 \end{bmatrix}.$$

Find bases for the row, column, and null spaces of A . Why does A not have any eigenvalues or eigenvectors?

- Work through the proof questions from the recent homework assignments (that is, the questions toward the end of each section). There will be proofs on the exam!

- Write down the matrices for the following linear transformations of \mathbb{R}^2 .
Hint: What happens to the standard basis vectors?
 - Rotation by an angle θ , counterclockwise.
 - Rotation by 45° clockwise, reflection over the line $y = x$, followed by rotation by 45° counterclockwise. Could you have ignored the rotations altogether and just reflected over a different line instead?
 - D rotates a vector 45° clockwise, stretches the y -component by factor of 3 and the x -component by $\frac{1}{2}$, then rotates 45° counterclockwise. Could you have ignored the rotations altogether and just stretched along different lines instead?
 - What are the eigenvalues and eigenvectors for the above transformations?
- Definition check. What are the mathematical definitions of “basis” and “dimension”? (Look these words up and copy down the definitions)
- Write down from memory the Fundamental Theorem. Here’s some help:
 - The five parts (a)-(e) deal with systems of linear equations and row reduction.
 - The two parts (f)-(g) deal with dimensions of important subspaces of A .
 - The three parts (h)-(j) deal with the columns of A .
 - The three parts (k)-(m) deal with the rows of A .
 - The two parts (n)-(o) deal with determinants and eigenvalues.

- Here are two matrices:

$$B = \begin{bmatrix} 3 & 4 & -1 \\ -14 & -6 & 1 \\ -52 & -20 & 3 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

- Are B or C invertible? If so, find the inverse using Gauss-Jordan elimination (see Examples 3.30 and 3.31 on pages 173-5)
- Row reduce B and C .
- Find bases for the row, column, and null spaces for B and C .
- Calculate the determinants of B and C .
- Find all eigenvalues and eigenvectors for B and C .
- Find bases for the eigenspaces of B and C . What are the algebraic and geometric multiplicities of the eigenvalues?
- How could the Fundamental Theorem have simplified many of the above calculations? Where would it not have helped?

Similarity and diagonalizability will not be on Exam 3 but they will be on the final, so try to answer these questions:

- Do the eigenvectors span \mathbb{R}^3 ? That is, are there three linearly independent eigenvectors?
- Are B and C similar matrices? Why or why not? (Hint: Theorem 4.22)
- Are B and C diagonalizable? (Hint: Look at the definition of “diagonalizable,” as well as Theorems 4.23 and 4.25.)