$$f(x) = 2x^3 - 3x^2 - 12x + 2$$

- $\lim_{x \to +\infty} f(x) = +\infty \qquad \lim_{x \to -\infty} f(x) = -\infty$

- $f'(x) = 6x^2 - 6x - 12$

  $f'(x) = 0 \iff x^2 - x - 2 = 0$

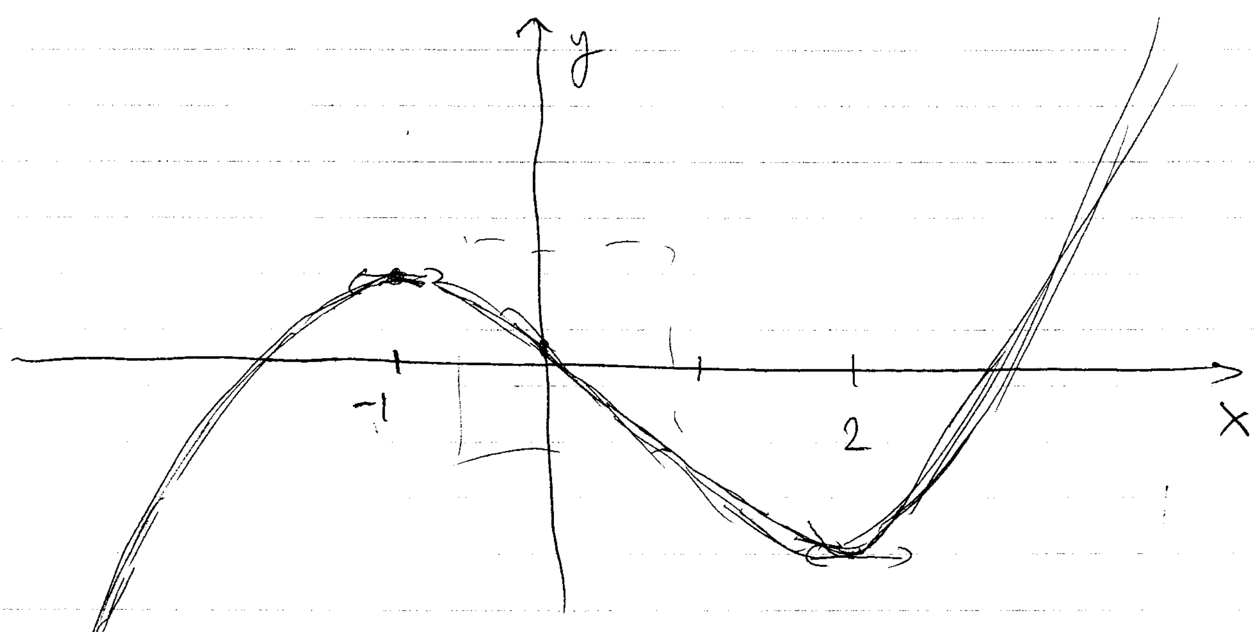
  $\iff (x+1)(x-2) = 0$

  $\iff x = -1 \text{ or } x = 2$

- $f''(x) = 12x - 6$

  $f''(-1) = -12 - 6 = -18 < 0$

  $f''(2) = 18 > 0$

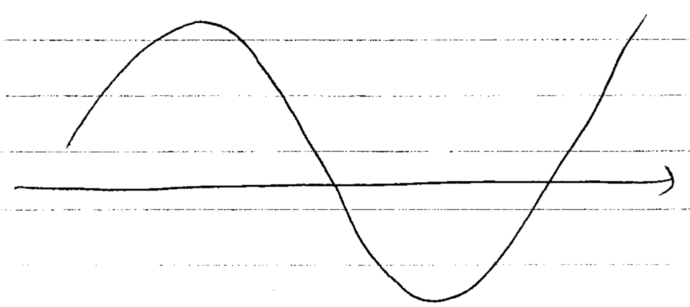


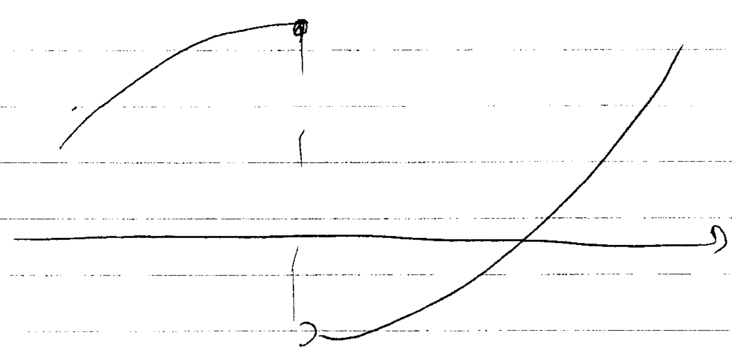
$$f(-1) = -2 - 3 + 12 + 2 = 9$$

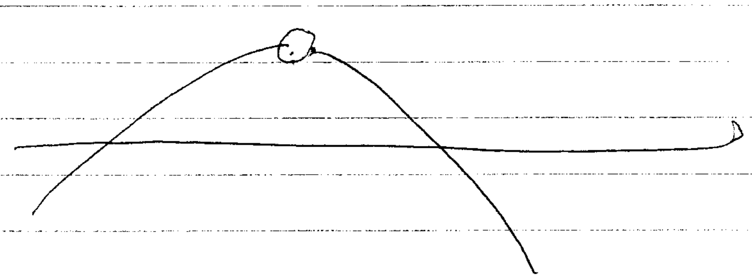
$$f(2) = 16 - 12 - 24 + 2$$
$$= -18$$

$$f(0) = 2$$

Continuity







For a function to be continuous, it has to be defined.

<u>Definition:</u> f is continuous at $x = x_0$ if and only if

$$\begin{cases} \lim_{x \to x_0^-} f(x) = f(x_0) \\ \lim_{x \to x_0^+} f(x) = f(x_0) \end{cases}$$

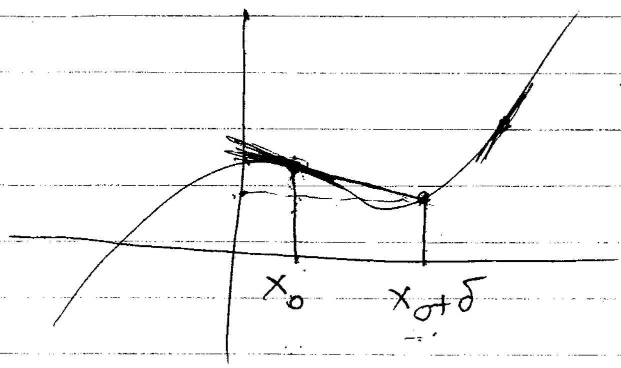
$$\lim_{x \to x_0} f(x) = f(x_0)$$

To translate the graph of f horizontally, set

$$\tilde{f}(x) = f(x - x_0)$$ where x_0 is a given number.

It is possible for a function to be continuous but not differentiable. <u>Ex</u> $f(x) = |x|$

Definition: f is differentiable at $x = x_0$ if & only if $\lim_{\delta \to 0} \frac{f(x_0+\delta) - f(x_0)}{\delta}$ exists



Then $f'(x_0)$ = is this limit.

Assume f is differentiable at x_0. We want to show that f is continuous at x_0, i.e. that

$$\lim_{\delta \to 0} f(x_0+\delta) = f(x_0)$$

$$\lim_{\delta \to 0} |f(x_0+\delta) - f(x_0)|$$
$$= \lim_{\delta \to 0} \left(\left| \frac{f(x_0+\delta) - f(x_0)}{\delta} \right| \cdot |\delta| \right)$$
$$= \lim_{\delta \to 0} \left| \frac{f(x_0+\delta) - f(x_0)}{\delta} \right| \cdot \lim_{\delta \to 0} |\delta|$$
$$= |f'(x_0)| \cdot 0 = 0$$