8/29/08

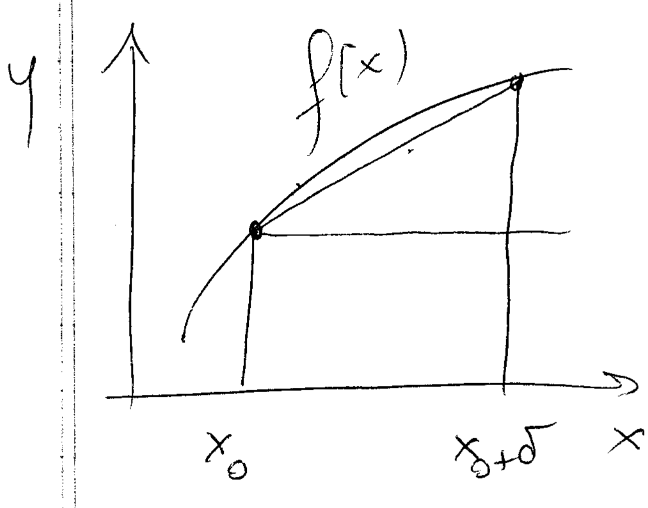
Continuity & differentiability (continued)

if $\lim\limits_{\delta \to 0} |f(x_0) - f(x_0+\delta)| = 0$

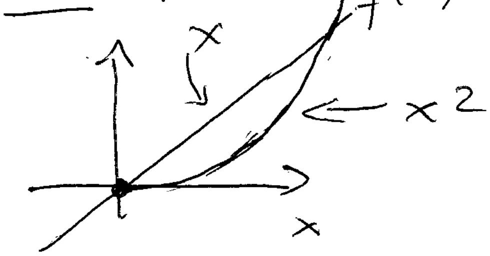
then f is continuous at x_0

if $\lim\limits_{\delta \to 0} \left[\dfrac{f(x_0+\delta) - f(x_0)}{\delta}\right]$ exists $(= f'(x_0))$

then f is differentiable at x_0



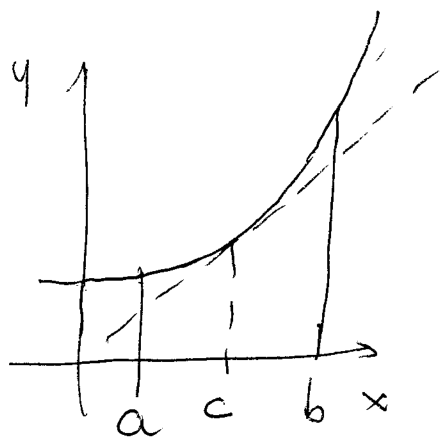
Ex Take $f(x) = x$



As $x \to 0$ $f(x) \to 0$

Look at $\dfrac{f(x)}{x} = \dfrac{x}{x} = 1$

Look at $\left|\dfrac{f(x)}{x^2}\right| \to \infty$ as $x \to 0$



Mean-value theorem: If f is (continuous) and differentiable on $[a,b]$, then there exists a value c, $a \leq c \leq b$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Local & global extrema

Definition: a point x_0 is a global maximum of a function f if $f(x_0) \geq f(x)$ for every x in the domain of definition of f.

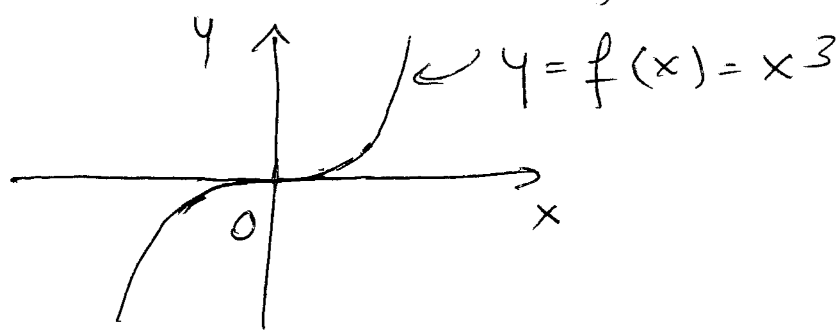
- It is possible for a function to have a local minimum at $x=0$ but not have $f'(x)=0$.

Ex:
- $|x|$
- $f(x) = x$ on $[0,1]$

- It is possible to have $f'(x_0) = 0$ and x_0 not being a local maximum or minimum of f.

<u>Ex</u>: $f(x) = x^3$ at $x = 0$

$f'(x) = 3x^2$ $\quad f'(0) = 0$



- It is possible for $f''(x_0)$ to be 0 and for f not to have an inflection point at x_0

<u>Ex</u>: $f(x) = x^4$ at $x = 0$

$f'(x) = 4x^3$ $\quad f''(x) = 12x^2$ $\quad f''(0) = 0$