9/24/08
Math 250A

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HISTORY OF INTEGRATION (to provide a bit of perspective)

⇝ our textbook represents the collective work of countless people stretching centuries back, some of the key players:

- <u>Archimedes of Syracuse</u> - (~~St~~ 200 BC) student of Euclid, method of quadratures to find areas, but could only be used in cases w/ a high degree of geometric symmetry

- <u>Isaac Newton / Gottfried Leibniz</u> (British) (German) (1600's) - each independently developed differential calculus and subsequently that integration allowed you to go 'backwards'
  - ⇝ we still use their notation

- <u>Augustin Louis Cauchy</u> (early 1800's) - French mathmetician who developed the theory of **limits**, formalizing the notion of an integral

- <u>Bernhard Riemann</u> (mid-1800's) - German, presented the idea of fillin[g] the area underneath a curve w/ little rectangles and taking the limit where the rectangle width becomes infiminte[simal]

- <u>Henri Lebesgue</u> (late 1800's - early 1900's) - French, proposed a method different from Riemann (both methods are unique but still complementary)

- <u>notation</u> - Leibniz first used the symbol $\int$, which is supposed to be a lon[g] and ~~stands for the~~ indicates that the integral is a limit of

Integration by Parts (IBPs)

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product rule: $\frac{d}{dx}(uv) = u'v + uv'$

$$(uv)' = u'v + uv' \xrightarrow{\text{rearrange}} uv' = (uv)' - u'v$$

now integrate both sides (w/ respect to x)

$$\int uv'\,dx = \int (uv')\,dx - \int u'v\,dx$$

$$= uv - \int u'v\,dx$$

extra const. of integration gets lumped into here

$$(uv)' = \frac{d}{dx}(uv)$$
$$(uv)'\,dx = d(uv)$$

$$\boxed{\int uv'\,dx = uv - \int u'v\,dx}$$

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ex $\int x \sin x \, dx$

$u = x \longrightarrow u' = 1$
$v' = \sin x \qquad v = -\cos x$

$$= -x\cos x - \int 1 \cdot -\cos x \, dx$$

$$= -x\cos x + \int \cos x \, dx$$

$$= -x\cos x + \sin x + C$$

How to choose u and v':

1. make sure you can figure out what v is from v'
2. it generally helps if u' is simpler than u
3. it helps if v is simpler than v'

Goal make $\int u'v\,dx$ a 'simpler' integral than $\int uv'\,dx$

$$\int uv'\,dx = uv - \int u'v\,dx$$

ex $\int x \ln x \, dx$

$u = \ln x, \quad du = \frac{1}{x}dx$

$\longrightarrow dx = x\,du$

$= \int x\,u\,x\,du = \int x^2 u\,du$

$\int x \ln x \, dx$

$u = \ln x \longrightarrow u' = \frac{1}{x}$

$v' = x \qquad v = \frac{1}{2}x^2$

$= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx$

$= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x\,dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

$= \frac{1}{2}x^2\left[\ln x - \frac{1}{2}\right] + C$

$\boxed{ex}$ $\int \ln x \, dx$ $u = \ln x \longrightarrow u' = \frac{1}{x}$

$v' = 1 \qquad v = x$

$= \int 1 \cdot \ln x \, dx$

$= x \ln x - \int \frac{1}{x} \cdot x \, dx = x \ln x - x + C$

$\boxed{\text{Definite Integrals}}$

$\int_0^1 x e^{-x} \, dx$ evaluate this part @ the bounds too! $u = x \longrightarrow u' = 1$

$v' = e^{-x} \qquad v = -e^{-x}$

$= -xe^{-x}\Big|_0^1 - \int_0^1 -e^{-x} \, dx$

$= -xe^{-x}\Big|_0^1 - e^{-x}\Big|_0^1 = 1 - \frac{2}{e}$

$\boxed{ex}$ $\int x^2 e^x \, dx$ $u = x^2 \longrightarrow u' = 2x$

$v' = e^x \qquad v = e^x$

$= x^2 e^x - \int 2xe^x \, dx$ [need to use IBPs twice!]

$\alpha = x \longrightarrow \alpha' = 1$

$= x^2 e^x - 2\left[xe^x - \int e^x \, dx\right]$ $\beta' = e^x \qquad \beta = e^x$

be careful not to choose same variable names

$= x^2 e^x - 2xe^x + 2e^x + C$

$= e^x \left[x^2 - 2x + 2\right] + C$

ex $\int e^x \cos x \, dx$ $\qquad u = \cos x \longrightarrow u' = -\sin x$

$\qquad v' = e^x \qquad v = e^x$

$= e^x \cos x + \int \sin x \, e^x \, dx$

$\qquad \alpha = \sin x \longrightarrow \alpha' = \cos x$

$= e^x \cos x + \left[e^x \sin x - \int \cos x \, e^x \, dx \right] \qquad \beta' = e^x \qquad \beta = e^x$

$\longrightarrow 2 \int e^x \cos x \, dx = e^x \cos x + e^x \sin x + C$

$\Longrightarrow \int e^x \cos x \, dx = \frac{1}{2} e^x \left[\cos x + \sin x \right] + C \quad \therefore$

* this problem shows that sometimes using IBPs, you can end up w/ the integral you started with, but you still have all the pieces you need to still write down an explicit solution