Equations of the form $y' = g(y)$ (continued)

Example: $N' = -N \ln(N)$ $N > 0$

Method of solution:

$$F(N) = \int \frac{dN}{N\ln(N)} = -\int dt = -t + C$$

Let $u = \ln(N)$ $du = \frac{dN}{N}$

$$\int \frac{dN}{N\ln(N)} = \int \frac{du}{u} = \ln|u| + \tilde{C}$$
$$= \ln|\ln(N)| + \tilde{C}$$

So $-t + C = \ln|\ln[N]| + \tilde{C}$

$$\Rightarrow -t + \tilde{K} = \ln|\ln(N)|$$

$$\Rightarrow \ln(N) = e^{-t + \tilde{K}}$$

$$\Rightarrow N = e^{e^{-t + K}}$$

Note: $F(N) = -t + C$ is the same

as $t = -F(N) + C$

<u>Singular solutions?</u>

$$N' = \begin{cases} 0 & \text{if } N = 0 \\ -N \ln(N) & N > 0 \end{cases}$$

Note $\lim_{N \to 0^+} N \ln(N) = 0$

So $N = 1$ is a singular solution

$N=0$ & $N=1$ are equilibria

$$\begin{cases} N=0 \text{ is unstable} \\ N=1 \text{ is stable} \end{cases}$$

If we set $N'=0$ if $N=0$ as above, then $N=0$ is also a singular solution.

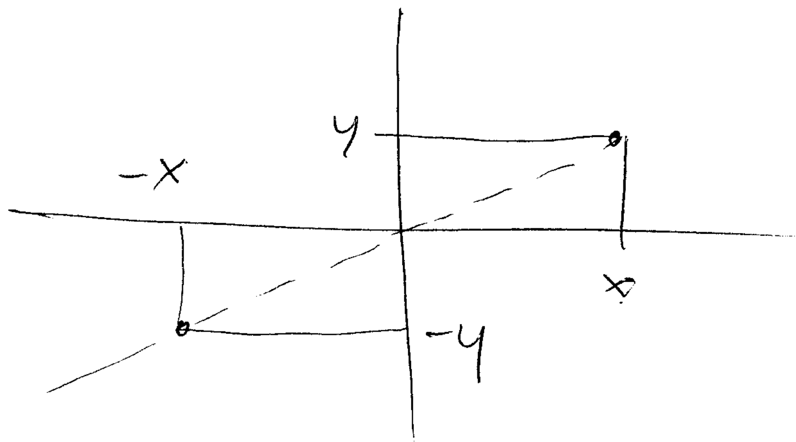
Aside on symmetries

$$\frac{dy}{dx} = g(y)$$

If g is even, i.e. $g(-y) = g(y)$

Change $y \to -y$ and $x \to -x$

The ode does not change



The family of solution curves is symmetric with respect to the origin.

If g is odd, i.e. $g(-y) = -g(y)$

Change $y \to -y$

$$\frac{dy}{dx} = g(y)$$

The family of solution curves is symmetric with respect to the x-axis.