

Equations of the form $y' = g(y)$
(continued)

The logistic equation

$$\frac{dy}{dt} = y(1-y)$$

1. Solve i.e. find the general solution:

$$\frac{dy}{y(1-y)} = dt$$

Integrate $t + C = \int \frac{dy}{y(1-y)} = - \int \frac{dy}{y(y-1)}$

With table V.26 $a=0, b=1$
we have

$$\begin{aligned} \int \frac{dy}{y(y-1)} &= -(\ln|y| - \ln|y-1|) + \tilde{C} \\ &= \ln\left|\frac{y-1}{y}\right| + \tilde{C} \end{aligned}$$

$$\begin{aligned} \text{So } t + C &= -\ln\left|\frac{y-1}{y}\right| + \tilde{C} = \ln\left|\frac{y}{y-1}\right| + \tilde{C} \\ \left|\frac{y}{y-1}\right| &= e^{t+C-\tilde{C}} \Rightarrow \frac{y}{y-1} = \pm e^{t+C-\tilde{C}} \\ &= Ke^t \quad K \neq 0 \end{aligned}$$

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$$\text{So } y = (y-1) e^t k = y e^t k - e^t k$$

$$\Rightarrow y (1 - k e^t) = - e^t k$$

$$\Rightarrow y = \frac{k e^t}{k e^t - 1} = \frac{1}{1 - \frac{1}{k} e^t} \quad (k \neq 0)$$

As $t \rightarrow +\infty$, $y(t) \rightarrow 1$ stable

$t \rightarrow -\infty$, $y(t) \rightarrow 0$ unstable