

Equations of the form $y' = g(y)$ (continued)

Phase lines and bifurcation diagrams

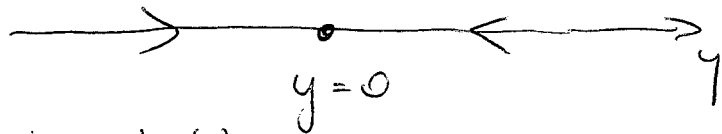
Example: $\frac{dy}{dt} = \mu y - y^3$

Equilibria: solve $\mu y - y^3 = 0$

$$\mu y - y^3 = 0 \Leftrightarrow y(\mu - y^2) = 0$$

$$\Leftrightarrow y = 0 \text{ or } y^2 = \mu$$

$\mu < 0$

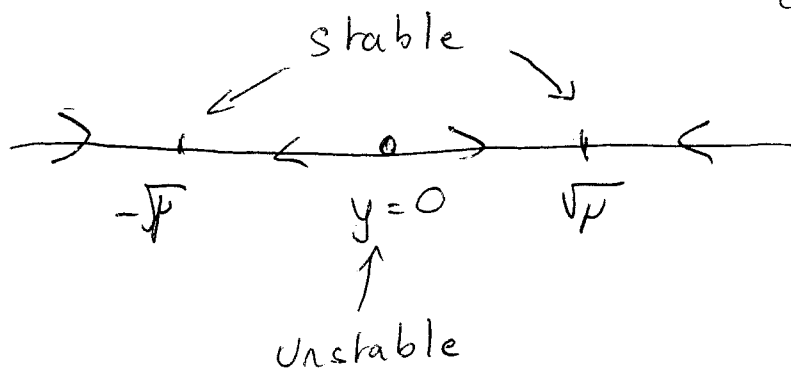


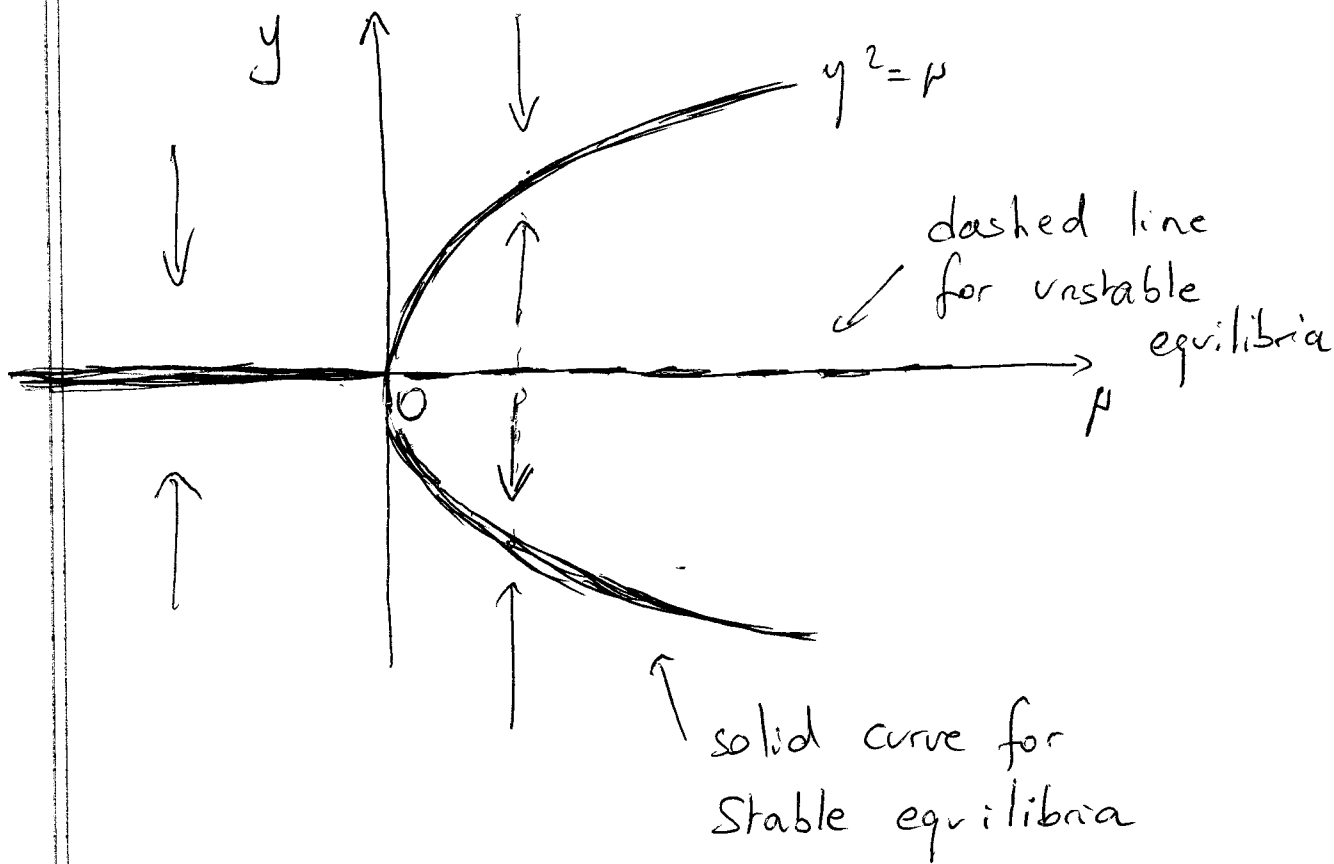
$y = 0$ is stable

$\mu > 0$

$y = 0$ $y^2 = \mu$

correspond to
equilibria



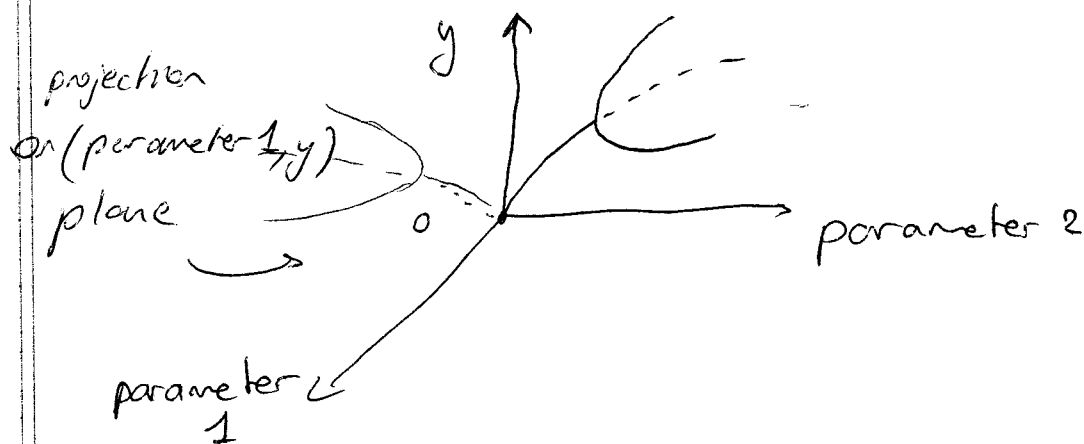


This is called a bifurcation diagram

A bifurcation occurs at $\mu = 0$

It is called a pitchfork bifurcation

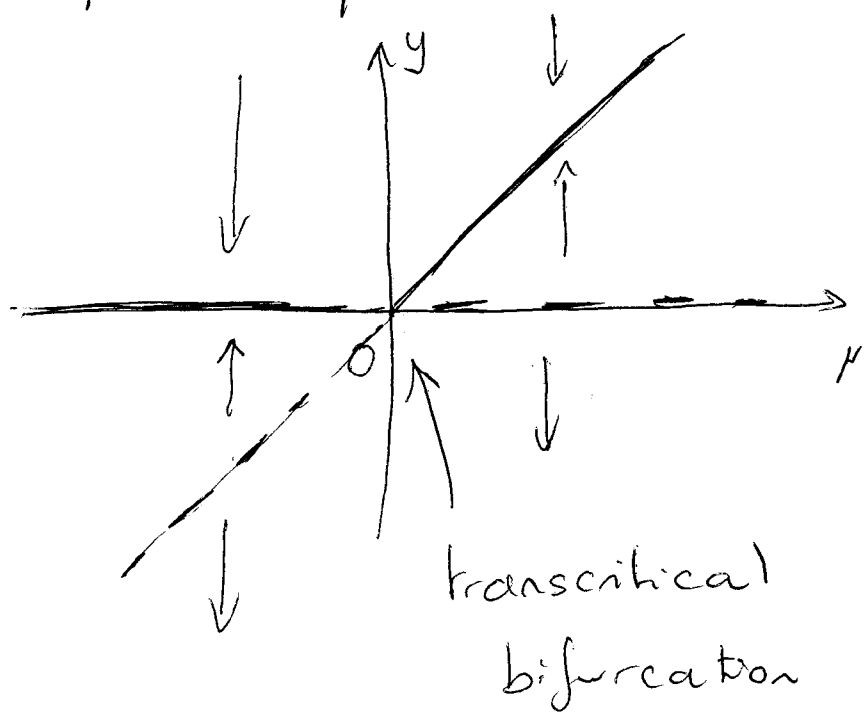
If there is more than 1 parameter



Example 2: $\frac{dy}{dt} = \mu y - y^2 = y(\mu - y)$

Equilibria: $y(\mu - y) = 0 \Leftrightarrow \begin{cases} y = 0 \\ \text{or } y = \mu \end{cases}$

Always 2 equilibria.



Example 3: $\frac{dy}{dt} = \mu - y^2$

Equilibria: $\mu - y^2 = 0 \Leftrightarrow y = \pm \sqrt{\mu}$

