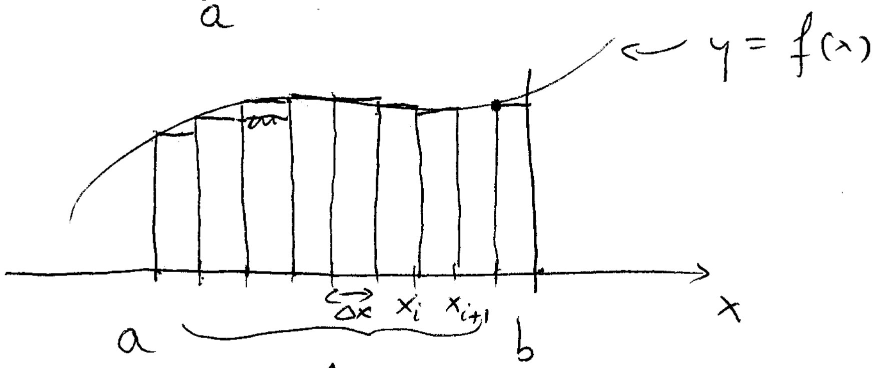
Numerical Approximations

$$I = \int_a^b f(x)\,dx \qquad \text{definite integral}$$



n subintervals of length $\Delta x = \dfrac{b-a}{n}$

$$x_i = a + i\,\Delta x \qquad i = 0, \ldots, n$$

Riemann sums:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=0}^{n-1} f(x_i)\,\Delta x \qquad \text{(left point)}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x \qquad \text{(right point)}$$

<u>Midpoint rule:</u> Use the value of f at the point in the middle of the interval $[x_i\ x_{i+1}]$.

Underestimates or overestimates?

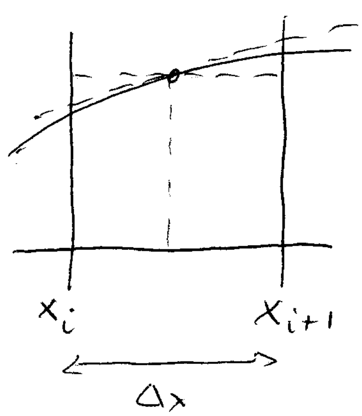
If f is increasing on $[a,b]$, then

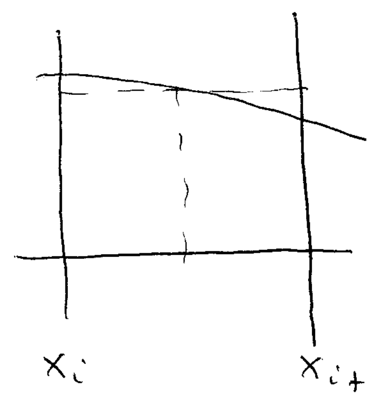
$$\text{LEFT}(n) \leq \int_a^b f(x)\,dx \leq \text{RIGHT}(n)$$

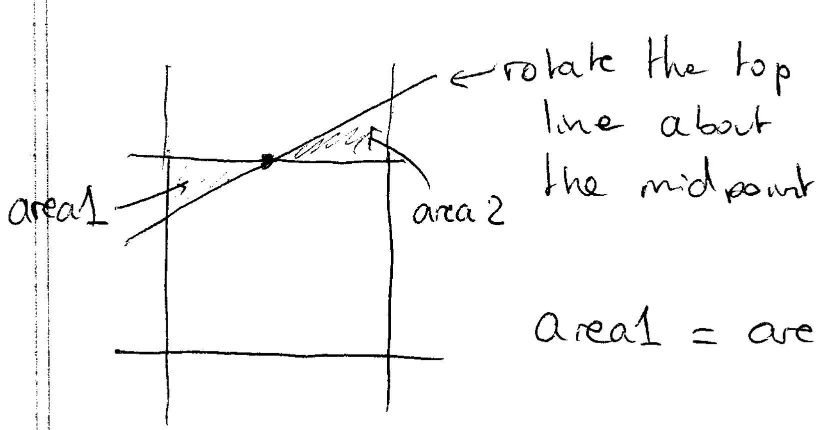
If f is decreasing on $[a,b]$, then

$$\text{RIGHT}(n) \leq \int_a^b f(x)\,dx \leq \text{LEFT}(n)$$

What about the midpoint rule?

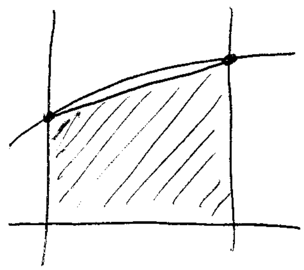


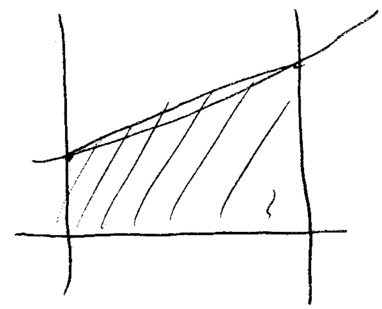




area1 = area2

What about the trapezoid rule?





If f is concave up

$$MID(n) \leq \int_a^b f(x)\,dx \leq TRAP(n)$$

If f is concave down

$$TRAP(n) \leq \int_a^b f(x)\,dx \leq MID(n)$$

Example:

f is positive, decreasing & concave down

positive
$$\begin{cases} \text{decreasing: } & RIGHT(n) \leq \int_a^b f(x)\,dx \leq LEFT(n) \\ \text{concave down: } & TRAP(n) \leq \int_a^b f(x)\,dx \leq MID(n) \end{cases}$$

$RIGHT(n) = 0.703$, $LEFT(n) = 0.745$

$$TRAP(n) = \frac{LEFT(n) + RIGHT(n)}{2} = 0.724$$