

11/21/08

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Numerical Approximations (continued)

The logistic map

$$x_{n+1} = a x_n (1 - x_n) = a f(x_n)$$

$$f(x) = a x (1 - x) = a (x - x^2)$$

Maximum of f : $f'(x) = a(1 - 2x)$

maximum at $x = \frac{1}{2}$

$$f\left(\frac{1}{2}\right) = a \frac{1}{2} \left(1 - \frac{1}{2}\right) = \frac{a}{4}$$

For iterates to remain between 0 & 1, we want $a \leq 4$.

Equilibrium points

We want $f(x) = x \Leftrightarrow a x (1 - x) = x$

i.e. $x = 0$ or $a(1 - x) = 1 \Leftrightarrow x = 1 - \frac{1}{a}$