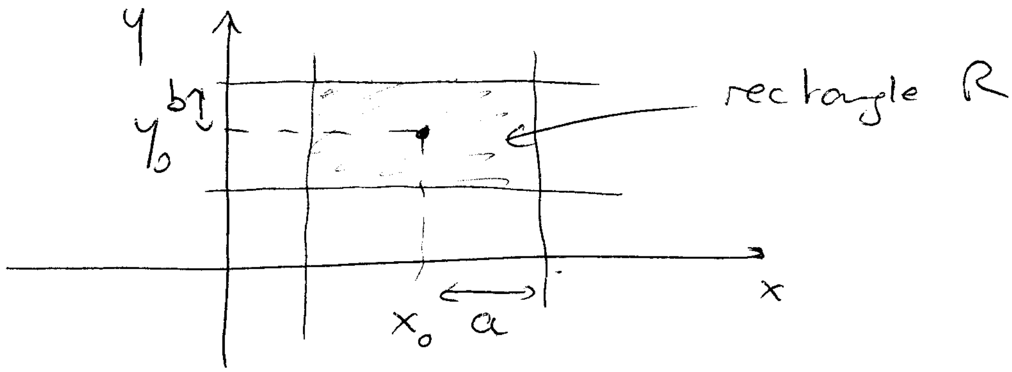
Equations of the form $y' = g(x, y)$ (continued)

Existence & uniqueness theorems

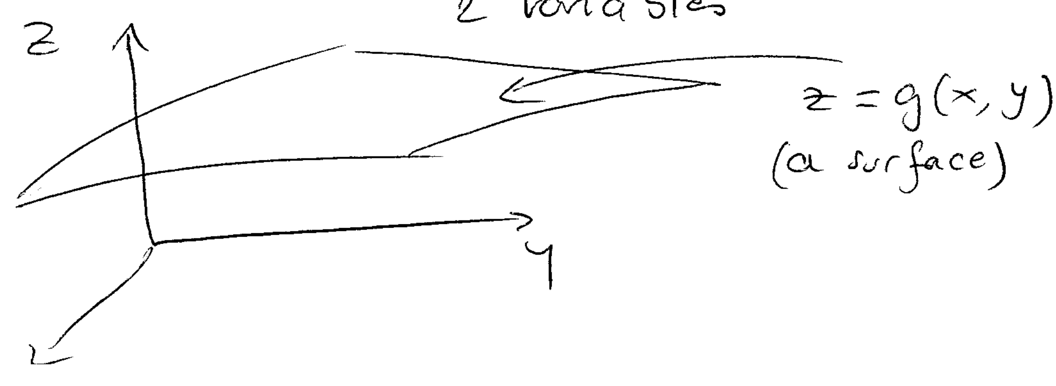


Continuity of $g(x,y)$ in R
$\Rightarrow$ existence of a solution "near" $\begin{pmatrix} x = x_0, \\ y = y_0 \end{pmatrix}$

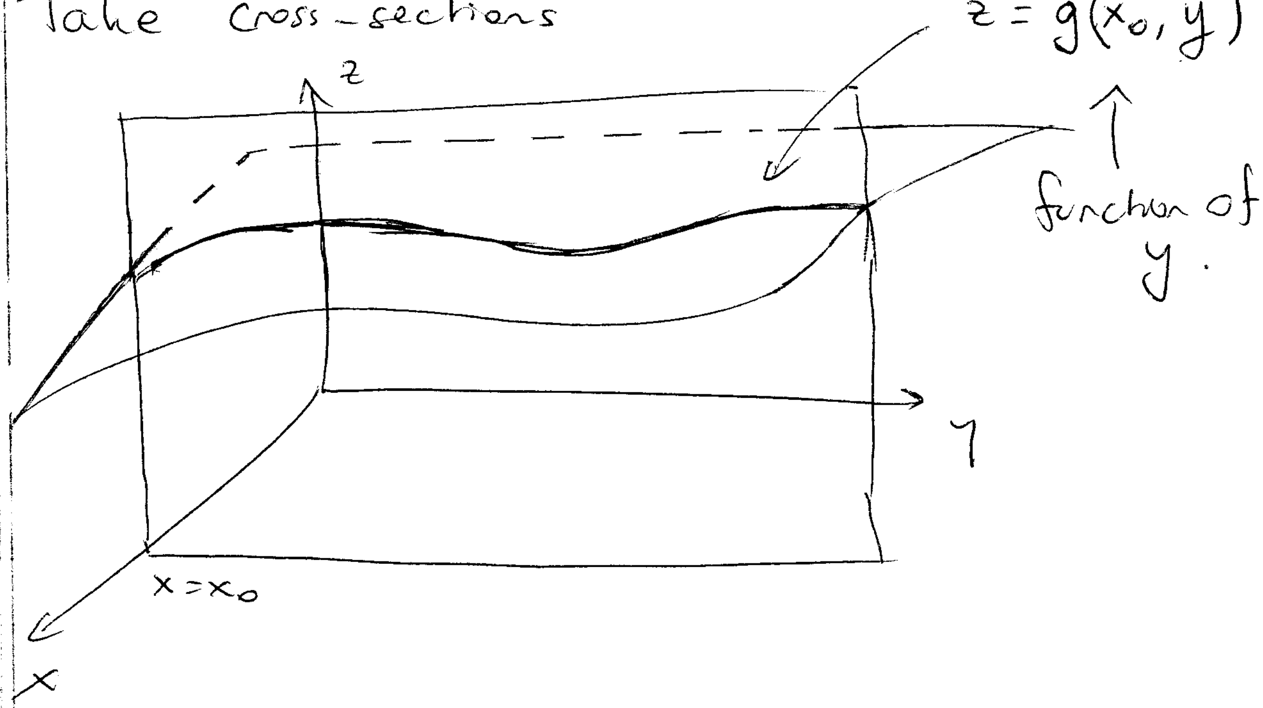
but not necessarily everywhere.

Uniqueness: Find $\frac{\partial}{\partial y} g(x,y)$ & check that it is continuous on R.

Partial derivatives: $g(x,y)$ is a function of 2 variables



Take cross-sections



Partial derivative of g with respect to y is denoted by

$$\frac{\partial g}{\partial y}(x_0, y) = \frac{d}{dy} g(x_0, y)$$

$$\frac{\partial g}{\partial y}(x_0, y_0) = \lim_{y \to y_0} \frac{g(x_0, y) - g(x_0, y_0)}{y - y_0}$$

<u>Example</u>: $g(x, y) = x y^2$

$$\frac{\partial g}{\partial y}(x_0, y_0) = \lim_{y \to y_0} \frac{g(x_0, y) - g(x_0, y_0)}{y - y_0}$$

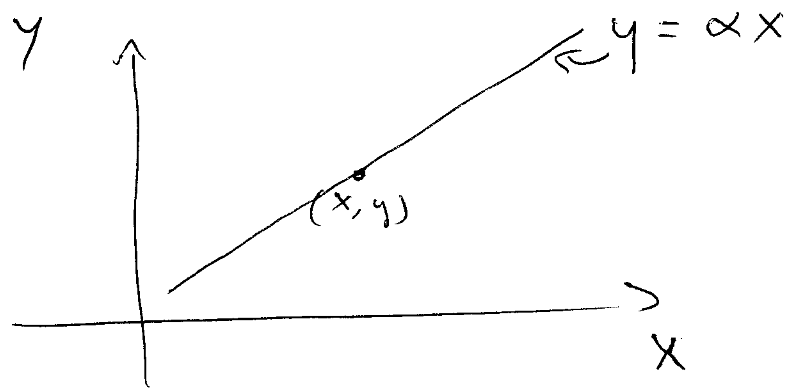
$$= \lim_{y \to y_0} \frac{x_0 y^2 - x_0 y_0^2}{y - y_0} = \lim_{y \to y_0} \frac{x_0 (y^2 - y_0^2)}{y - y_0}$$

$$= \lim_{y \to y_0} \left[x_0 (y + y_0)\right] = 2 x_0 y_0$$

So $\frac{\partial}{\partial y}(x y^2) = 2 x y$

<u>Example</u>: without using the limit

$$\frac{\partial}{\partial y}\left(\sin(x)\cos(y)\right) = -\sin(x)\sin(y)$$



Look at function with $y = \alpha x$

$\sin(x)\sin(y) = \sin(x)\sin(\alpha x)$
& ask whether the function is continuous in x, for every α.