Calculus and Differential Equations I

MATH 250 A

Maxima, minima, inflection points, and differentiability

Sketching the graph of a function

$$f(x) = 2x^3 - 3x^2 - 12x + 2$$

- Do you know how to sketch the graph of this function without using a calculator?
  1. Yes
  2. No
- If one adds a constant to the right hand side of the above equation, what does this do to the graph of f?
  1. Translate the graph horizontally
  2. Translate the graph vertically
- What do you need to do to translate the graph horizontally?

Continuity and differentiability

- Can a function f be continuous but not differentiable at a point?
  1. Yes
  2. No
- Can a function f be differentiable but not continuous at a point?
  1. Yes
  2. No
- If f is a smooth function, is it possible to have f' positive and f locally decreasing?
  1. Yes
  2. No
- If f is a smooth function of x and $f''(x) > 0$, then
  1. f is concave up
  2. f is concave down
  3. One cannot tell

Local and global extrema

- A function f has a local maximum at a point x_0 if $f(x) \leq f(x_0)$ for x near x_0.
- Similarly, a function f has a local minimum at a point x_0 if $f(x) \geq f(x_0)$ for x near x_0.
- An extremum (minimum or maximum) of a function can be local or global.
- Does a function always have a global maximum?
  1. Yes
  2. No
- Is it possible for a function to have more than one global minimum?
  1. Yes
  2. No

Extrema and first derivative

- If a function f has a local minimum at $x = 0$, does it necessarily mean that $f'(0) = 0$?
  1. Yes
  2. No
- A function f has a critical point at $x = x_0$ if either $f'(x_0) = 0$ or $f'(x_0)$ is undefined.
- Fact: If f has a local maximum or minimum at $x = x_0$, if x_0 is not an end-point of the domain of definition of f, and if f is differentiable at $x = x_0$, then $f'(x_0) = 0$.
- We now consider the converse of this statement. What should we require of f' for f to have a local maximum or a local minimum at $x = x_0$?

Extrema and first derivative (continued)

- If f is differentiable and $f'(x_0) = 0$, does it mean that f has a local maximum or minimum at $x = x_0$?
  1. Yes
  2. No
- First-derivative test: if f has a critical point at $x = x_0$, is continuous at $x = x_0$, and is differentiable on the right and on the left of x_0, then
  - If f' goes from negative to positive as x is increased past x_0, then f has a local minimum at $x = x_0$.
  - If f' goes from positive to negative as x is increased past x_0, then f has a local maximum at $x = x_0$.
- In particular, if f is twice differentiable on some interval, if f'' has a constant sign near x_0, and if x_0 is not an end-point of the interval, then f has a local maximum or minimum at $x = x_0$ if and only if $f'(x_0) = 0$.

Extrema and second derivative, inflection points

- Second-derivative test: If f is twice-differentiable, if f'' is continuous, and if $f'(x_0) = 0$, then
  - If $f''(x_0) > 0$, then f has a local minimum at x_0.
  - If $f''(x_0) < 0$, then f has a local maximum at x_0.
  - If $f''(x_0) = 0$, then one cannot conclude the existence of a local extremum at $x = x_0$.
- An inflection point of a function f is a point x_0 such that the graph of f changes concavity at $x = x_0$.
- In other words, the graph of the function f crosses its tangent at an inflection point.
- How are inflection points and second derivatives related?

Inflection points and second derivative

- If f is twice-differentiable and x_0 is not an end-point of the domain of definition of f, is the following statement true of false?

  Statement: If $f''(x_0) = 0$, then f has an inflection point at $x = x_0$.
  1. True
  2. False
- If f is twice-differentiable and if f'' changes sign when x goes through x_0, then f has an inflection point at $x = x_0$.
- Reading assignment: Section 4.1 of the *Calculus* book.
- Homework: *Review of differentiation* in WebAssign.