We start with:

\[ m\ddot{x} + b\dot{x} + kx = 0 \quad m, b, k > 0 \quad \dot{x} = \frac{dx}{dt} \text{ and } \ddot{x} = \frac{dv}{dt} \]

From physics we know \( F = ma \) and since \( a = \ddot{x} \) this becomes \( F = m \ddot{x} \) and in a spring, which is the case we investigated, \( F = -kx \)

Looking at the homogeneous case of this would give us:

\[ m\ddot{x} + kx = 0 \quad \text{and} \quad x(t) = A \cos(\omega t + \varphi) \]
\[ -\omega^2 A \cos(\omega t + \varphi) = -\frac{k}{m} A \cos(\omega t + \varphi) \]
\[ \omega = \sqrt{\frac{k}{m}} \text{ and this is the Resonant frequency} \]

Now we go to the Non-homogeneous case.

\[ m\ddot{x} + kx = F(t) = F_0 \cos(\omega t) \text{ but this time } x(t) = B \cos(\omega t) \]
\[ -\omega^2 m B \cos(\omega t) + k B \cos(\omega t) = F_0 \cos(\omega t) \]
\[ B(-\omega^2 m + k) = F_0 \text{ the cosines cancel and B factors} \]
\[ B = \frac{F_0}{-w^2 m + k} = \frac{F_0}{-w^2 m + k} = \frac{F_0}{Bm(k - \frac{w^2}{m})} = \frac{F_0}{Bm(w_0^2 - w^2)} \]

Now looking at the graphs

**Graph 1:**
- \( B = \frac{F_0}{k} \)
- \( \omega = \sqrt{\frac{k}{m}} \)

**Graph 2:**
- \( x(t) = k \cos(\omega t + \varphi) \)
- \( \varphi = \pi \)
- \( w_0 \)
- \( \omega \)

**Natural Frequency**
Now we look again at the equation

\[ m\ddot{x} + kx = F(t) = F_0 \cos(\omega t) \] but this time \( x(t) = Be^{\lambda t} \)

\[ \lambda = \frac{-b \pm \sqrt{b^2 - 4(m)(k)}}{2m} \]

Look at the particular solution

\[ z(t) = Ae^{i(\omega t - \delta)} \quad \dot{z} = Ai\omega e^{i(\omega t - \delta)} \quad \ddot{z}(t) = -A\omega^2 e^{i(\omega t - \delta)} \]

We plug these back into the equation to get

\[ -mA\omega^2 e^{i(\omega t - \delta)} + bAi\omega e^{i(\omega t - \delta)} + kAe^{i(\omega t - \delta)} = F_0 e^{i\omega t} \]

Simplifying gives us the equation

\[ F_0 = Ae^{-\delta t}[-m\omega^2 + bi\omega + k] \]

\[ A(\omega) = \frac{F_0}{m} \]

\[ \delta = \tan^{-1}\frac{b\omega}{m(\omega_0^2 - \omega^2)} \]

These are the graphs for the above 2 equations

This all can also be used for understanding of electrical concepts since mechanical and electrical have a very close correlation. Many terms can be related between the two.

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