1. Consider the following differential equation:

$$\frac{dx}{dt} = \lambda\, x - \gamma\, x^3$$

   a. What is the dimension of λ?
   b. What is the dimension of γ? Your answer should be in terms of the dimension of x, $[x]$.
   c. Let t_0 be a characteristic time scale for this problem. Define a dimensionless time variable $\tau = t / t_0$, and show that you can change variables from t to τ, to "get rid" of the parameter λ.
   d. Can you find a change of variable that would allow you to "get rid" of γ as well?

2. The force of gravity between two bodies of mass m_1 and m_2 has intensity $F = G\frac{m_1 m_2}{r^2}$, where r is the distance between the centers of mass of the two bodies, and G is the gravitational constant.
   a. Use this formula to show that for an object of mass m at the surface of the Earth, one can approximate the force F by $F \cong m\, g$, where the acceleration of gravity g is constant.
   b. Express g in terms of G, the mass M of the Earth, and the radius R of the Earth.

3. Consider a smooth function $f(x)$, and its expansion near $x = x_0$,

$\sum_{i=0}^{n} f^{(i)}(x)\frac{(x-x_0)^i}{i!} + f^{(n+1)}(\bar{x})\frac{(x-x_0)^{n+1}}{(n+1)!}$ where $\bar{x} \in [x_0, x]$, and call $E_n(x)$ the error made by approximating f with its Taylor expansion truncated to order n,

$$E_n(x) = f(x) - \sum_{i=0}^{n} f^{(i)}(x)\frac{(x-x_0)^i}{i!}.$$

   a. Apply this formula to $f(x) = \cos(x)$, with $x_0 = 0$ and $n = 5$.
   b. Find a condition on $|x|$ which ensures that $|E_n(x)| < 0.05$.
   c. Use a calculator or Matlab to check your answer to the previous question.