

Read the article by R. May entitled *Simple mathematical models with very complicated dynamics* (Nature **261**, 459-467 (1976); there is a link to the online version of this paper on the course assignment page), and address (i.e. explain, justifies, work out the details of, prove) the following statements found in this paper.

1. Page 460, left column, above Equation (3): *By writing $X = bN/a$, the equation may be brought into the canonical form*

$$X_{t+1} = aX_t(1 - X_t).$$

2. Page 460, left column, bottom: *If X ever exceeds unity, subsequent iterations diverge towards $-\infty$.*
3. Page 460, right column, below Equation (6): *So long as this slope lies between 45° and -45° ... the equilibrium point X^* will be at least locally stable.*
4. Page 461, left column, below Equation (9): *Clearly, the equilibrium point X^* of Equation (5) is a solution of Equation (9).*
5. Equation 10:

$$\lambda^{(2)}(X^*) = [\lambda^{(1)}(X^*)]^2$$

6. Page 461, left column, bottom: *As this happens, the curve $F^{(2)}(X)$ must develop a "loop", and two new fixed points of period 2 appear.*
7. Page 461, left column, bottom: *This slope is easily shown to be the same at both points, and more generally to be the same at all k points on a period k cycle.*
8. Bottom of page 461 and beginning of page 462: *... until at last the three-point cycle appears (at $a = 3.8284...$ for equation (3)).*
9. Page 462, right column, bottom: *This period 3 cycle is never stable.*
10. Page 462, right column, bottom: *As $F(X)$ continues to steepen, the slope $\lambda^{(3)}$ for this initially stable three-point cycle decreases beyond -1 ; the cycle becomes unstable and gives rise by bifurcation process ... to stable cycles of period 6, 12, 24, ..., 3×2^n .*
11. Draw pictures illustrating the concepts of tangent and pitchfork bifurcations (see page 463, top of left column).
12. Page 464, right column, top: *... the slope of the k -time iterated map $F^{(k)}$ at any point on a period k cycle is simply equal to the product of the slopes of $F(X)$ at each of the points X_k^* on this cycle.*

13. Page 465, left column, bottom: ... as each new pair of cycles is born by tangent bifurcation (see Fig. 5), one of them is at first stable, by virtue of the way smoothly rounded hills and valleys intercept the 45° line.
14. Page 466, right column, bottom: ... in continuous two-dimensional systems ... dynamic trajectories cannot cross each other.