PEER INSTRUCTION IN PHYSICS AND MATHEMATICS

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ABSTRACT: We explore a method of teaching called Peer Instruction. We begin by describing how Peer Instruction was implemented in physics and summarizing the results and then discuss the way in which Peer Instruction was modified to be used in an introductory single variable calculus course (first and second semesters) and summarize those results.

KEYWORDS: Calculus, peer instruction, mathematics education, ConcepTest, conceptual calculus, innovative teaching in calculus.

DESCRIPTION AND IMPLEMENTATION

Peer Instruction was developed by Eric Mazur, a physicist at Harvard University,[1] in response to the results of the Force Concept Inventory (FCI) test. This multiple-choice exam tests physics students' understanding of the conceptual ideas behind Newtonian mechanics. After two months of instruction, students appeared able to solve standard problems in class, but lacked the ability to answer some basic conceptual questions on the FCI test.[1, p. 15] This led to the question, "Are students learning physics or symbol manipulation?"

Mazur addressed this issue by creating and implementing what he called "ConcepTests". Using these ConcepTests, the student's role was transformed from that of passive observer to active participant.

A ConcepTest is a conceptual question that is presented with one correct answer and other incorrect answers in a multiple-choice format. For example, "A person tries to knock down a large wooden bowling pin by throwing a ball at it. The person has two balls of equal size and mass, one made of rubber and the other of putty. The rubber ball bounces back while the ball

of putty sticks to the pin. Which ball is most likely to topple the pin? 1) the rubber ball 2) the ball of putty 3) makes no difference 4) need more information."[1, p. 131] While the solution to this question can be attained through the application of equations derived from conservation of momentum, it is intended that the student arrive at the answer by considering the idea of impulse and Newton's third law, 'To every action there is an equal and opposite reaction'.

Peer Instruction was implemented in physics by using four ConcepTests in every 60-minute period, breaking the class into four sections of 15 minutes apiece. The first 7-8 minutes of each section would be used to introduce the students to the ideas behind certain physical principles. For the example above, Newton's third law, impulse and conservation of momentum would be explored with minimal use of equations. Then the ConcepTest would be presented to the students in transparency form. The students would be given one minute of silence during which they arrive at an answer individually. The students demonstrate their answers by voting. At this point, if a significant percentage of the students have chosen the correct answer ($\sim$50 to 80%), the students are given one to two minutes to discuss the options and possibly convince others of their own thinking. This step is a critical part of Peer Instruction. After discussion the students once again vote for the option they now think is correct. (Usually a significant increase in correct responses is recorded.) The fifteen-minute period ends with the instructor explaining the correct answer. These conceptual classes constituted all of the lectures when done at Harvard. The problem solving was left to discussion classes.

In order to create an environment where the students will properly invest themselves in Peer Instruction, two measures were taken: reading assignments and conceptual questions on the exams. The reading assignments required students to read certain sections of the book, on which they will be quizzed, in preparation for what will be covered in class. The conceptual questions on the exams were similar to those done in class and required a multiple-choice answer along with a paragraph explaining the answer chosen. Usually these conceptual questions constituted 50% of the credit on an exam. The remaining credit came from more traditional problems solved using equations.

RESULTS

The effectiveness of Peer Instruction has been measured in two ways: performance on conceptual problems and performance on standard problems, as measured against a standard lecture class. The FCI test was used to evaluate conceptual understanding, while the Mechanics Baseline test was used to evaluate standard problem solving skills. After two months of standard (non-Peer Instruction) lecture, the 1990 Harvard introductory physics class (121 students) scored an average of 78% on the FCI test.[1] After two months of Peer Instruction the 1991 (177 students), 1993 (158 students), 1994 (216 students), and 1995 (181 students) classes scored respectively; 85%, 86%, 88% and 88%. The mechanics baseline test yielded the following scores for the appropriate years: 1990-67%, 1991-72%, 1993-73%, 1994-76%, and 1995-76%.[1,p.16] These results show a dramatic improvement in the answering of conceptual questions along with a marked improvement in standard problem solving ability. "Apparently, and perhaps not surprisingly, a better understanding of the underlying concepts leads to improved performance on conventional problems."[1, p. 16] Since Mazur's first implementation of peer instruction at Harvard, the method has spread to other universities, colleges and community colleges throughout the country.

Having taught introductory college physics using both standard lecture and Peer Instruction, I can personally attest to results similar to Mazur's, along with an increase in students' confidence, retention, alertness and enthusiasm.

PEER INSTRUCTION IN CALCULUS

Description and Implementation

This past year I taught the first two semesters of introductory calculus using Peer Instruction adapted for this purpose. I followed a procedure very similar to that used in physics. The differences occurred in the following areas, listed in order of most significant to least: the ConcepTests, the reading assignments, and the grading of the conceptual problems on exams.

It was necessary to create a year's worth of ConcepTests. While only three are presented here, I have made over 150 ConcepTests that deal with calculus on a conceptual level. Using my Tuesday/Thursday class (eighty minutes apiece) as an example, the entire Tuesday class and thirty minutes of the Thursday class was used for ConcepTests while the remaining class time was spent on problem solving.

Consider that the normal physics ConcepTest deals with words, images, and objects with which the students already have an inherent familiarity.

[1]This can be compared to the pre-class average of 70% for the FCI, based on the years 1991-1995.

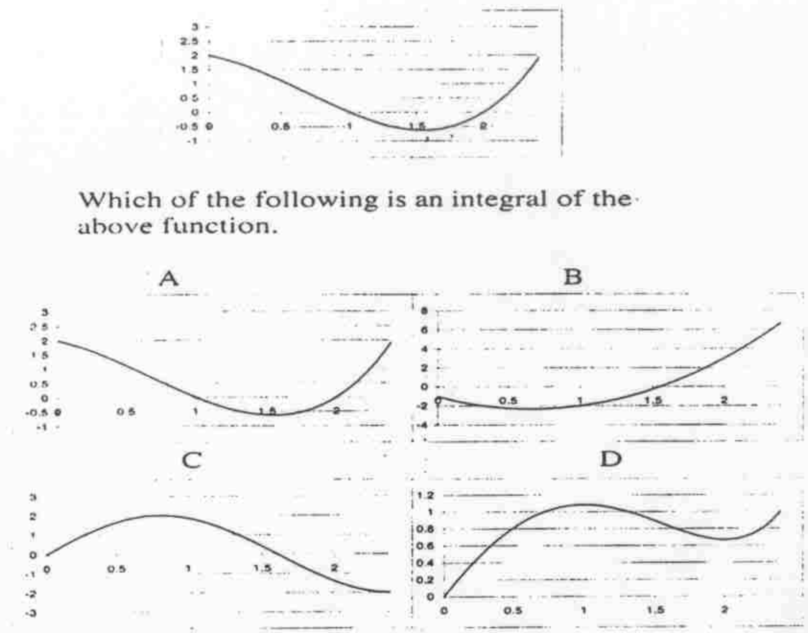

Figure 1. This ConceptTest was used to probe the concept of "area as a function of x."

This is not the case in calculus. It was for this reason that given one central idea, I felt the need to develop and use several ConcepTests that started simply and increased in complexity. For example, take the concept of the derivative, where each ConcepTest question focused on, "Which of the following graphs represents the derivative of the above function?" The first ConcepTest presented along these lines was made to be as simple as possible, $F(x) = x$ or $F(x) = x^2$, where the subsequent ConcepTests would build in complexity to functions that would be hard to identify in graphic form. After the techniques of analysis were established to some extent via the first ConcepTest, the subsequent ConcepTests were used to probe the ideas more deeply.

A similar example involves Taylor Series. The first ConcepTest put forward on this subject presented the graphs of, $F(x) = x$ or $F(x) = x^2$. The students were asked the question, "Which of the following graphs represents the addition of these two functions?" This paved the way for more complex questions dealing with the addition of off-centered parabolas and higher order polynomials. (See Figure 3.)

In physics the reading assignments involved the text. Given the nature of the calculus book I used this past year, Larson, Hostetler and Edwards' sixth edition of *Calculus* [2], I felt that reading the text on ideas not yet covered in class would be too intimidating for the students. It is for this reason that I altered the reading assignments. Given a certain section of the book with 9 worked examples, I chose 5 or 6 examples for the students to study, 1 or 2 of which would be asked on a quiz in the following class. The examples asked on the quiz were identical to those done in the book. Other methods of quizzing could be used given a text that explored the conceptual ideas more fully and from a point of view more accessible to the students.

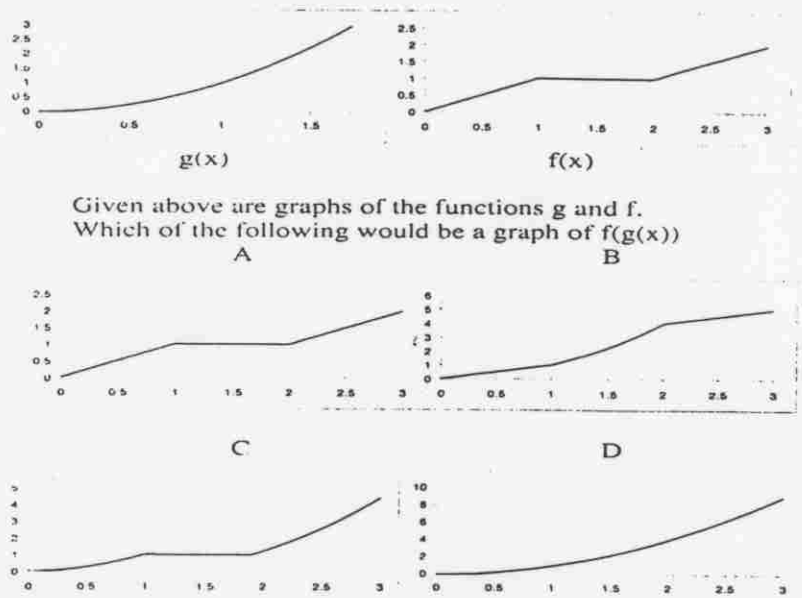

Figure 2. Composition of functions was used to develop the idea behind the chain rule.

While using Peer Instruction in either physics or calculus, it is important to stress to the students that their work on the concepts will be tested in the exams. In calculus versus physics, it was necessary to go to greater lengths in emphasizing the importance of the discussion over just picking the correct answer. For this reason I made the students aware, from the beginning, that picking the correct answer on an exam would yield only 1 out of 10 points. The rest of the points would come from the written justification.

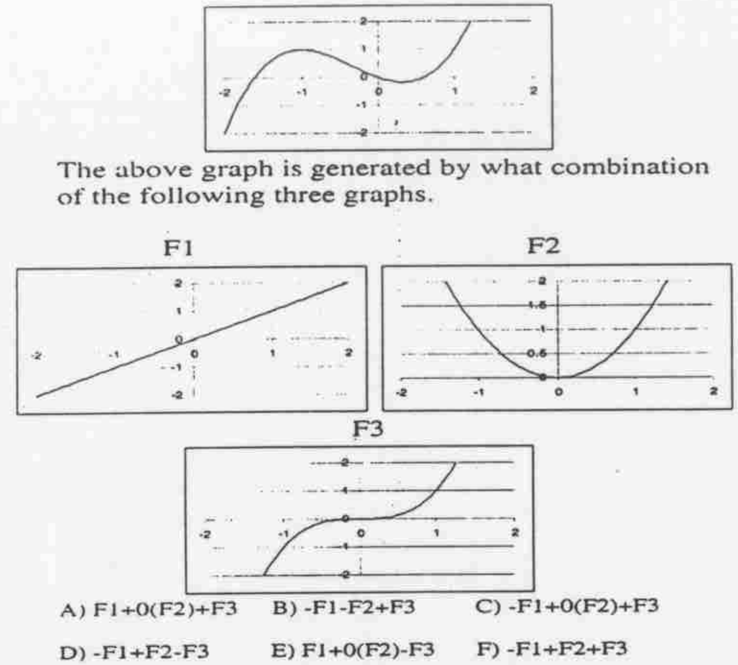

Figure 3. The graphical addition of functions was used to introduce the Taylor series.

It proved useful in both semesters to hand out a syllabus that had all reading assignments, homework assignments, and tests laid out from the beginning. The reading assignments were critical for keeping the class on schedule. For both semesters, a non-trivial portion of the grade came from the reading quizzes.

RESULTS

After two semesters of calculus Peer Instruction, I can attest to results similar to those I have had with Peer Instruction in physics. Over the period of a year, the students showed a significant improvement in reasoning skills. This was accompanied by impressive class retention. This retention was demonstrated in the review for the final. In this three-hour review, the students were presented with every ConcepTest done in the semester. For each ConcepTest presented, over ninety percent of the students chose the correct answer. More important than reasoning skills and retention, an actual enthusiasm for calculus grew in many of the students. Students also showed the same kind of confidence I have seen in physics Peer Instruction. The only complaint I heard from the students was that they had to read material before we covered it in class. Students have registered similar complaints in physics. The students, however, could not deny the benefits of the reading assignments. I felt that the reading assignments were critical in establishing a link between the conceptual ideas and their specific symbolic representation.

These Peer Instruction classes, I was told by the head of the mathematics department at Albright College, contributed significantly to the number of new mathematics majors. Several students were self-motivated to seek out members of the administration to deliver praise for this method. Even in consideration of these advantages, the most important to me was the daily feedback that I gained from the students through the answers to the ConcepTests. This information was invaluable to me in planning and teaching classes.

Specific results were gained at the beginning of the second semester, when my two classes consisted of students instructed via peer instruction for the first semester (17 students) and students instructed in a more traditional lecture (13 students). The first day of class, which was one and a half months after finals of the last semester, I gave all of my students a test over first semester material that consisted of three conceptual problems and two standard problems. Students instructed via standard lecture averaged 17% on the conceptual and 54% on the standard problems, while those taught via peer instruction averaged 73% on the conceptual and 63% on the standard problems.[2] It is also noteworthy that simpler conceptual problems were offered at reduced credit. These included the graphic identification of the derivative of the function $f(x) = x^2$. 2 out of 13 students instructed via standard lecture were able to identify the derivative of $f(x) = x^2$. Due to several factors, such as the small number of students, these specific results can not be accepted as proof of this method. They do, however, suggest a promising avenue to explore, especially since the results follow similar trends to those produced by Mazur.

I have included three ConcepTests, (see Figures 1, 2 and 3). Each of these ConcepTests is of middle difficulty in their individual sequence of ConcepTests. Figure 1 investigates the students' understanding of the integral as an "area as a function of x", including negative regions. Figure 2 is one of a sequence of ConcepTests that developed the ideas behind the chain rule. Figure 3 is part of a sequence of ConcepTests that leads to Taylor Series.

[2]Student identities were unknown during grading. In the conceptual questions, points were given solely on the basis of picking the right answer, no partial credit was given for the written answers.

ACKNOWLEDGMENTS

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BIOGRAPHICAL SKETCH

Scott Pilzer received his PhD in Physics in 1997 from Lehigh University. He worked with his advisor, Dr. Beal Fowler, on explaining certain energy transfer phenomenon using analytical models in quantum mechanics. Since his graduation he has had the opportunity to teach physics at Utah State University, Santa Clara University, Southern Utah University, and Albright College, while following his wife's career. He currently lives in Reading PA with his wife and two dogs, where he teaches Calculus for Albright College.