A Document on Some Aspects of the Educational Mission of a Small College Mathematics Department in 1990

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In 1990, the Calculus Reform Movement was gaining momentum. A few mathematicians had expressed the feeling that something was wrong in college math education, and they were determined to do something about it; a larger number were aware of these concerns, but were not actively involved. The vast majority appeared uninterested.

During the decade of the 1990’s, the situation changed. Evidence of the magnitude of the transformation can be found by looking at the programs for the joint American Mathematical Society-Mathematics Association of America (AMS-MAA) meetings that occur every January. Special sessions devoted to educational issues have become common, and many are sponsored by the AMS. A decade ago, this did not happen.

What brought about the change? I think there were two big events in the 1990’s that forced math departments to look at their educational mission very carefully. First, the job market went through the worst period since the early ’70’s. Unemployment for new Ph.D.’s was close to 15% in one year. For a long time, it had been anticipated that the ’90’s would be decade of high demand as the hires of the ’60’s passed into retirement. These hopes, however, were dashed as university departments contracted under tighter budgets. The second event was a national re-orientation of the scientific research goals. The signal event came in 1994, when the Senate committee chaired by Barbara Mikulski that was in charge of the NSF budget directed the Foundation to support “strategic” research (read “economically meaningful”) in preference to “curiosity-driven” research. In the following years, the rhetoric was toned down, but the change had been made. In 1996, the University of Rochester briefly considered eliminating its mathematics graduate program—another challenge to the values of traditional mathematical research, with its “other-worldly” orientation.

By this time, it was perfectly clear that mathematics departments could not continue “business as usual”—that is, business as defined in the 1960’s. Both these events forced mathematicians to consider what society at large valued in the work of mathematicians. Two directions were clear. One way that mathematicians could prove their worth would be by aiding other researchers, particularly those who visibly contributed to the material welfare of society. Another way would be by addressing educational concerns. There was a note of materialism here, too. Arguments for supporting education in the 1990’s were often based on the economic rewards that accompanied better education.

The 1990’s were a decade of profound change. Hyman Bass (the current president of the AMS) said that mathematics was going through a “phase change”—and alteration as profound as a change from liquid to solid. It is not yet clear what the new equilibrium will be. What new goals will mathematics departments assimilate in their self-image? We do not know.

At this time, it is illuminating to look back on the conditions 10 years ago. How far have we come? I wrote the following document for a talk I gave at the University of Illinois Chicago Circle in the spring of 1990. It reviews the state of affairs, as I then saw them, at Indiana University South Bend. The document is pretty much self-contained. I have made
a couple of editorial changes, and have added some commentary in sections 7 and 8, which were never written. As I reread it, I think of the phrase, “Things change, and yet remain the same.” But rather than commenting on the document in this introduction, I will let it speak for itself.

From among the lessons I have learned in a decade of involvement with the issues this document raises, there is one I want to single out. In 1990, I expressed concerns about the effect of relying on “adjuncts” to deliver instruction; see §3. At the time, I tended to view this group as somewhat monolithic—though I did acknowledge that there were differences. What I would like to emphasize after 10 years of experience is the range and diversity of talent, commitment, drive, creativity, etc. that is to be found among the non-professorial instructors. Institutions would do well to seek ways to tap into this resource.

Reforming Math Education at IUSB
James J. Madden, Indiana University South Bend, May 7, 1990

Abstract. Remedial, business math and elementary teacher preparatory courses offered at Indiana University South Bend have tended to have overly routine and mathematically trivial contents. Some of the institutional forces which in the past have pushed these courses in this direction are described. At present, efforts to restore mathematical vitality are underway. Examples of course materials from a geometry class designed to involve students more actively and meaningfully are presented. Some factors interfering with change are mentioned.

0. Introduction.
When I attempt to speak about reforming mathematics education, I am much aware of the impossibility of speaking with the assurance I’m used to in my field of research—mathematics. I cannot guarantee, or even confidently suggest, anything of the form “if such and such is done then such and such an educational outcome will be achieved.” I know that good serious research on educational strategies is being conducted and that the results can be applied. In matters like this, I’m probably no better informed than many of you. Even so, the classroom is not a laboratory, and the outcomes are always somewhat uncertain. Moreover, the most important aims of education are years beyond the completion of schooling, and success with these cannot be quantified.

My concern with issues in mathematics education springs from personal experience in chance circumstances. During the years I’ve been at IUSB, the department has faced a number of challenging problems relating to its teaching mission. These have included questions concerning the effectiveness of remedial, service, and teacher education courses. There has been a serious problem concerning student preparation. There have been questions about staffing. In addition, there are issues raised by attempts to implement the newly formulated educational goals, e.g., the new NCTM standards and the ideas proposed by the “Calculus Reform Movement”. What I want to talk about are some of the things which have most troubled my colleagues and myself, the remedies we tried, the directions we envisioned and the obstacles we encountered.
IUSB has offered a microcosm of many of the educational problems facing math departments in larger universities around this nation. In exploring avenues for reform there, it has become clear that much more is involved than simply changing classroom practices and goals. Math education is a social institution, not a technology. Those who want to improve undergraduate math education must also deal with the system that selects students and instructors, equips them with norms and values, sets their goals, determines the setting in which they are brought together, tells them what to do vis-a-vis one another and establishes rewards. Thinking on this level is a matter of practical necessity because situations beyond the classroom will influence instructional design and impinge on our effectiveness as educators. Frankly, I would not hold high hopes for any movement to improve education that was not bold enough look beyond curriculum changes and take on the challenges of improving the institutions that deliver instruction.

1. Indiana University at South Bend.

Let me begin by telling you some things about the institution where I teach. IUSB is a commuter campus currently serving just under 7,000 students. Only about a quarter of these are “traditional college students” entering almost directly from high school and falling in the 18-24 age group. The remainder comprise several “non-traditional” groupings, the largest of which are: 1) adults returning to college after interruptions of as much as 10 years, 2) graduate students and 3) special adult non-degree students (many of whom are exploring the possibility of college for the first time). Minorities are under-represented. Admissions to the university are non-selective. Students are required to take a mathematics placement test before enrolling for the first time in any math course. In pure math, we have 9 full-time faculty all of whom hold Ph.D.’s. Most have research interests; a few are very active in research.

In the fall of 1989, enrollments in mathematics courses totalled about 1150. About 950 were in courses below the freshman calculus level. Of the remaining 200, about half were in the ordinary first semester calculus course, 40 were in second semester calculus and the rest (60) were in various junior and senior courses. I think we have only one graduating math major this year. As you see, over 80% of the students serviced by the mathematics department were in “0- and 1-level courses”: remedial algebra, precalculus, finite math, business calculus, elementary school teacher training courses and “math-for-poets.” (As percents of math enrollments in Fall 1989, the figures were: remedial algebra, 23%; precalculus, 17%; finite math, 14%; business calculus, 12%; elementary school teacher training courses, 9%; “math-for-poets,” 5%). In all courses, we see large numbers of non-traditional students, but regrettably few minorities.

Of the “0- and 1-level courses,” about two thirds to three quarters of all sections are taught by adjuncts. These are individuals with college degrees (and often graduate degrees) in math or science. Many are local high school teachers. They are paid about $1200 per semester to teach a three-hour course.
3. Problems with the present math program.

Nearly a quarter of our enrollments are in remedial courses. Pre-calculus is really a preparatory course, too. Its largest consumer group is the business students whose placement tests are not strong enough to get them into Finite Math and Business Calculus. Thus, 40% of our enrollments are in preparatory courses.

I was amused to read in Thomas Clark’s history of Indiana University that dealing with inadequately prepared students was one of the major problems that David Starr Jordan confronted in his first years as university president in the 1880’s. Indiana University then was operating an extensive preparatory department and had been forced to lower the standard work of the university in order to have any students enrolled. In April, 1887, Jordan announced that he thought the preparatory school should be closed even if it did result in a reduction of enrollment. With the abolition of the preparatory school, the faculty recommended establishing close and friendly relations with Indiana high schools, and a program of visits to the high schools was begun. (Star of Jordan, where shine you now?)

The extent to which the IUSB Math Department ought to be involved in providing remediation has been a frequent subject of discussion. Our most elementary remedial course (M007, which covers what I was taught in 8th and 9th grade) is in the process of being handed over to a separate program (the Academic Resource Center). The other remedial course (M014, which covers what I was taught in 9th and some of what I was taught in 11th grade) may go the same way.

I am not satisfied that these courses, as we have been teaching them in the department, are as meaningful as they could be. Students are mainly being drilled in highly specific skills, particularly those that are easily testable. (E.g., solve $3x + 2 = 9x - 1$; graph $y + 5 = 3(x - 1)$; find the equation for the line through $(7,1)$ and $(6,4)$.) I have a feeling that students are not acquiring a sense of the meaningfulness of mathematics and are developing expectations about mathematics which are very damaging in subsequent courses. I’ll say more about this later.

The prospects that these courses will be handled better outside the math department are rather dim, I fear. The forces that push in the direction of routinization and trivialization are likely to be even stronger. I think it’s crucial for the math department to continue examining its goals for these courses and to stay involved with them, even if their administration is removed from the department.

The numbers I gave a short time ago make one of our most urgent problems apparent. Senior staff is trained and conditioned to respond to the needs of a population that makes up at most a mere 20% of our student clientele. Among full time faculty, I judge there to be significant enthusiasm for and devotion to teaching at the calculus level and above. There is generally a much lower level of interest in the 0- and 1-level courses. From time to time, service courses manifest symptoms of institutional neglect. For example, in 1988 we had to deal with vociferous complaints from large numbers of students in the elementary teacher training courses, counter-accusations by instructors, frayed tempers and hurt feelings.

It is perfectly clear to me that every faculty member really does care a great deal about their students. Every full-time faculty member teaches courses at all levels and everyone works hard to do it well. So when I talk about institutional neglect, I emphatically do
not want you to imagine any personal negligence or selfishness or anything like that. On the other hand, it is not easy to raise the level of concern among a majority to the point where people will sit down together and make fundamental decisions about these courses and take the time consuming steps necessary to see that these decisions area implemented in the classroom. It’s much easier to allow a structure to evolve impersonally and then teach within that frame when one’s turn to do so comes around.

A third problem source is the extensive use of faculty adjuncts. This is a very tricky thing to talk about, because people are going to think I’m saying that the adjuncts can’t teach as well as Ph. D. faculty. I don’t say that. And I don’t think it either. The adjunct’s job is to deliver a package that has been prepared (sometimes diligently, sometimes not) by the Mathematics Department. Their pay barely covers the time they devote to this, so they can’t be called on to contribute to design, which can be quite time consuming. The tendency is to provide them with routinized material in which minor adjustments are made from time to time. (I’d like to say that there are several adjuncts at IUSB who don’t fit the picture I’m painting. Two years ago, when we planned changes in remedial algebra, three of our adjuncts met with faculty regularly throughout an entire summer. These people collectively made major contributions to the design of two courses. Last semester, we formed a committee on the curriculum to which these same people were regular contributors. They didn’t receive any money for the extra time they gave us—but they won my admiration.)

All the factors I’ve discussed push in one basic direction: toward the routinization, ritualization and trivialization of classroom practice. The tendency is abetted by textbook publishers, who well understand the demand for labor-saving approaches to education, and to some extent by the students themselves, especially those who are attempting to satisfy formal prerequisites for programs outside math.

4. Formulating Goals

In the Fall of 1987 and Spring of 1988, a group of four faculty worked together to write a proposal to the NSF for a planning project to develop an improved calculus course. The exercise was extremely useful in terms of getting some clear statement of general goals which would guide future curriculum review, even though it did not contain any detailed specifications for any courses. In retrospect, we seem to have been responding to the problem of routinization.

Among the ideas suggested was a more topical approach to instruction, in which the same mathematical ideas would be presented many times at different levels of abstraction and rigor.

The funding that year was low and there was strong competition due to high level of interest in calculus reform. The proposal was not funded, and while some of us did attempt some revision in the way we taught calculus, nothing radical was tried.
5. Example: Geometry for Elementary Teachers.

As a service to the Education Department, IUSB Math teaches a series of three courses (called the “T-courses”) for intending elementary school teachers. As I mentioned above, these have been somewhat neglected in the past, and recently became a source of some considerable concern.

The first chance I had to experiment with a course that departed significantly from standard patterns was in one of these courses. In the fall of 1989, I had the opportunity to teach the third in the series, a course in basic geometry. In the past, various approaches were taken, including a sort of accelerated high school course involving proof writing. This seems to have been quite unpopular with students. After looking at Indiana competency guidelines for elementary teachers, I found there to be very little specific geometry content on which my students would be tested after leaving my course, so I felt quite free to depart from the syllabus we had been using.

Students in the T-courses frequently complain that the material they are presented has little relevance to teaching grade school children. Right or wrong, it was clear that this was a perception which would have to be dealt with head on. The first thing I decided about the course was that it should include a lot of material that could be translated into lessons for grade schoolers fairly directly. However, it was not to be limited to material of this kind. The second thing I decided about the course was that I wanted students to active. I wanted a lab-type atmosphere. The goals stated in the syllabus were:

1) To foster an appreciation for geometric thinking and to encourage independent exploration and discovery in geometry,
2) To equip prospective elementary teachers with the background and understanding necessary to make informed decisions about the elementary curriculum and classroom practice (insofar as geometry is involved),
3) To present a broad overview of some aspects of geometry (practical, theoretical, historical).

Geometry: An investigative approach by P. O’Daffer and S. Clemens and the accompanying workbook were nominal texts. The workbook eventually provided a good deal of very useful and appropriate material. The text itself helped to set the tone for the course and determine the order in which topics were considered, but was actually used very little.

The entire course lasted 14 weeks and met twice a week. The first six weeks were mainly devoted to topics related to polygonal figures, and culminated with a class on Fisk’s proof of Chvatal’s “Art Gallery Theorem,” (which offered the opportunity to review some of the main ideas of the first part of the course). The next five weeks were devoted to topics relating to symmetry and symmetry groups. The last three weeks were devoted to topics on measurement (especially of area).

Class format varied. On some occasions, I simply handed out materials and gave some brief directions. On other occasions, I talked and demonstrated (frequently at the overhead projector) for the whole class period (an hour and 15 minutes).

Here are some examples of things that were done:

1) On the first class, I brought a large (approximately 18 inch radius) plywood disc to class and challenged students to find the center. No one was able to suggest a way to find it, so I instructed students to continue thinking about the problem...
and bring a solution to class when they found one. Eventually, two young men suggested measuring the circumference, computing the radius and drawing two circles centered on the circumference of the plywood disc having the same radius as the disc. A young woman suggested making a paper tracing and folding it in half two ways. I provided the materials that were needed and let the students demonstrate their ideas in front of others. The class was asked to comment on the proposals, and to decide whether the procedures would, if executed with perfect precision, locate the center.

2) First exposure to triangles: Paper triangles of various shapes were provided to students, and they were asked to “make a generalization about the sum of the angles.” The immediate, and very reasonable, response was to ask what I meant by a sum of angles. After that was explained, several students tore the corners off their paper triangles and added the angles physically. Further questioning and folding led to a better approximation of a proof that the sum of the angles is a straight angle: one folds along the line joining the midpoints of two sides. Of course, there are a lot of things about triangles that must be either checked or assumed. Discussion of these points was encouraged.

3) Another activity with triangles: After being told the definitions of median, altitude, angle bisector and perpendicular bisector, students were asked to find ways of constructing these lines by folding paper triangles.

4) Some of the activities which in my opinion conveyed the most about the nature of mathematics involved the classification of things. What became my favorite exercise was something I would have predicted to be quite boring before I actually used it. It came from the workbook. Students were asked to classify all triangles and all quadrilaterals whose vertices were among the 9 lattice points \((i, j), i, j \in \{-1, 0, 1\}\). On another occasion, they were asked to classify all graphs on 5 vertices.

5) Students were required to do a project.

So what happened?

1) There was a generally high level of enthusiasm. (Respecting the students’ career goals seemed very valuable in promoting this.)

2) Different students participated at vastly different intellectual levels.

3) The quality of the projects I received varied greatly.

4) I didn’t have any good way of evaluating student work. Grades, ultimately, were simply based on attendance and quality of project.

5) I was uneasy concerning the question of how much content had actually been transmitted. There was a lack of standards of performance.

What effect would courses like this have on the teaching of math in the elementary schools? I see no reason to believe that this kind of experience, once finished, would produce a lasting difference in teaching practices of many of those who took it. However, if there were follow-up workshops to which students could return, there could be a real positive effect.

One might contrast the idea of the teacher education course as the occasion to fill up future teachers with the information that they will subsequently deliver to their pupils with
the idea of the course as an initiation into a life of mathematical activity. (This is close to some of the things mentioned in the calculus proposal.) However, I am now skeptical that all aspiring teachers can be set moving in this direction in the course of a couple of semesters. The type of projects which I received from my students suggests to me that in spite of all the enthusiasm I witnessed in the classroom, there are many people who still really would rather not deal with mathematical ideas on their own, independently and without someone to prod and encourage them. But with a sufficiently positive experience in the college classroom, they would, I think, be more than willing—perhaps even anxious—to return for more. And after a period of years (rather than weeks) of exposure, they might really become mathematical people.

Of course this also touches the theme of institutional change. The present system for installing teachers might not allow for this on a scale that would be socially meaningful. Creating the mechanisms that would renew contacts between university scholars and school teachers on a—let us say—monthly basis do not exist.


Now I want to consider an example that shows some ways in which institutional arrangements can impinge on a mathematics course in such a way as to inhibit meaningful learning and block positive change. At IUSB, we teach a course called “Applied Calculus.” This is a familiar course taught around the country. Mathematicians, with lack of affection, sometimes call it “Calculus for those who don’t want to know calculus.” The students who take it at IUSB are primarily intending business majors, for whom the course is required. Few, if any, take it by choice. I believe that the business school appreciates the filtering it achieves by requiring students to earn a C in the course as a condition for admission.

Enrollments average about 140 per semester, accounting for about 12% of all math enrollments at IUSB. Three to five sections are typically taught, one or two of which are staffed by Ph. D.-holding faculty, the others by “adjunct instructors.” The text is a variant of a forgotten ‘ur-text’ of a genus that first appeared at a punctuation in the evolution of calculus texts which occurred some time in the early ’70’s. It contains acceptable expositions of some basic calculus ideas, plus lots of recipes. The students learn the recipes.

You can tell from my tone that I consider the course a mathematical waste. Organizationally, however, it works like a clock. The adjuncts require little or no supervision, and no semester-by-semester planning or revision is required. Preparation for lessons requires little time. Students readily understand what is expected of them. The average ones get average grades with an average amount of study. With a little more work, the good ones get good grades. When the book is followed closely, there is a marvelous economy of effort. The publisher now supplies a bank of standardized tests which save even more time. The students like the tests because they don’t contain any surprises.

By the end of the semester, students have got some rules for differentiating memorized, they can find the maximum and minimum values of a cubic on a closed interval, they can figure out what rate of compound interest will make an investment double in nine years and they can find antiderivatives of polynomials. Most have great difficulty with word problems. No one, including me, has any idea what relevance any of this has to a career
in business, or anything else outside the classroom. Interestingly enough, the students for
the most part accept this. I believe many of them expect math to be a collection of odd
rules the reasons and rationale for which they are not intended to question. They view the
course as a hurdle they have to jump over to get the degree they want.

I’ve observed the way applied calculus courses are treated at other universities, and
some of the reasons for their widespread existence is clear. They provide a semblance of a
mathematics course while requiring minimum involvement from professors and students.
The textbooks are beautifully efficient in this regard. Who in his right mind would interfere
with a system that keeps so many people so quietly occupied for so much time?

I taught this course in the ritualized way in the Summer of 1989. By the usual
standards things went O.K. Students did well on the type of test provided by the publisher.
Course evaluations showed satisfaction with the intellectual content, and ranked the quality
of instruction somewhat above average. When I taught the course last semester, I tried
some experiments. I sought to encourage students to pay more attention to the “reasons
why” and less to the “how to.” I put less emphasis on learning specific procedures and told
students that I wanted them to work toward a level of understanding that would enable
them to devise appropriate strategies on their own. Test questions required more written
explanation and less plug and chug.

An odd phenomenon (which goes along with some of what I’ve been saying about
routine) occurred in these courses. Strangely, it was the weaker students who more readily
embraced to goals I was trying to market. Some of the most talented students seemed to
remain skeptical about my plans for them. One young woman wrote a note on an exam
complaining that it was “unfair.” But her score was near perfect and well above any other
in the class. Why would this happen? My hypothesis is that the good students master
a fairly straight forward strategy for success in algebra and precalculus courses. In vague
terms, it must amount to learning certain patterns in the book and in the lectures and then
reproducing them on tests. The strategy is inappropriate when test questions focus more
on ability to explain and extract meaning. When the strategy fails to produce the expected
results, this is going to lead to frustration and perhaps even the conviction that the teacher
is not doing things right. On the other hand, the weaker students have not constructed
very effective strategies in previous courses, and therefore don’t feel the frustration of not
being able to use what had previously been a ticket to success. (This assessment was
seconded by Armond Spencer of SUNY Potsdam who recently spent several days with us.
He had noted the same phenomenon in response to his own teaching.)

Generally, there was a high level of frustration with the tests I gave. Students felt, I
think, that the lectures and homework did not prepare them for the questions I was asking.
The students were right. While the lectures were aimed at exposing conceptual structure,
I don’t think the class was prepared to absorb the kind of material I was presenting. I was
able to get students to interact with me, but overestimated the significance. Toward the
end of the course, I found a strategy for lessons which worked fairly well. I would spend a
short time talking at the beginning of the class about whatever topic was scheduled. Then
I would pass out a “quiz” and move about the room helping students do the work. There
was not enough time to really develop this fully, so I plan to attempt something like this
from the beginning next time I have the opportunity.
In any case, I have a very vivid experience of how unsatisfactory the lecture format can be. Another practical lesson was how the text seriously interfered with any attempt to treat the elements of calculus in a thoughtful manner. I think the best thing to do would be to prepare a brief set of notes on the basic concepts and use this in conjunction with the lesson format just described.

On a more positive note, students ultimately did buy my point that learning plug and chug would not serve them well in life after college, and there was a readiness to try something more serious.

7. Attracting more majors: Lessons from Potsdam.
This section was never written. The small campus of the State University of New York at Potsdam was very successful in attracting a lot of math majors (20% of all graduates, I believe). I arranged a visit to IUSB by Dr. Armond Spences, who acted as a spokesman for the program. To the best of my recollection, the main points he made were as follows.

1) All faculty were “on the same wavelength” in terms of viewing their mission as teaching, and agreeing on some basic vision of what a math major should understand.

2) Faculty shared a clear sense of what mathematics is—it’s about proving theorems—and they paid attention to this from the start. This does not mean that students did proofs in calculus, but probably did mean that students were asked to think and analyze from the beginning. There was a lot of variety in the styles of teachers, but somehow they managed to keep a focus on a simple, central message.

3) Faculty spent a lot of time on the teaching task; and spent a lot of one-on-one time with students.

4) Graduates were successful getting jobs, partly because the program had a very excellent reputation among employers as a source of good employees. The department maintained ties with graduates; there was a bulletin board with letters from past graduates.

8. Conclusions.
This section was also never written. However, the manuscript shows that two sections were planned:

A) Institutional arrangements that benefit good teaching
B) “Jacksonian Math”

Both of these are discussed in the essay I contributed to the LaCEPT publication “Through the Eyes of Faculty.”