## ABSTRACTS

## Mini Courses:

- Sven Bachmann

  **Title:** Adiabatic dynamics and linear response theory for many-body quantum systems

  **Abstract:**The main theme of this lecture series is the understanding of the local non-autonomous dynamics of interacting, many-body quantum systems. When the driven dynamics is generated by a time-dependent family of gapped Hamiltonians, and the driving is slow, then an initial ground state is (parallel) transported within the ground state subspace up to controllable errors (adabatic theorem). As an application, we shall discuss a rigorous derivation of Kubos formula of linear response theory. If time permits, a proof of quantization of the Hall conductance for such interacting systems will be sketched.

- Fernando Brandao

  **Title:** Entanglement Theory

  **Abstract:** In these lectures we will cover the fundamentals of the theory of quantum entanglement. Topics include criterion of testing entaglement, distillability, and Bell inequalities. We will also discuss entanglement in the context of quantum many-body systems.

- Jan Philip Solovej

  **Title:** Quasi-free variational models for bosons and fermions

  **Abstract:** In these lectures we will discuss quasi-free states for bosons and fermions. Restricting to such states leads to variational models that approximate physical systems of bosons or fermions. For bosons this is what we call the Bogoliubov variational model. For fermions it is the Hartree-Fock-Bogoliubov model. We will discuss the mathematical theory of these models and show how they might be used to analyze physical phenomena such as superfluidity and superconductivity.

- Gunter Stolz

  **Title:** Disordered quantum spin chains

  **Abstract:** Spins are the simplest non-trivial example of a quantum particle. Thus quantum spin systems provide some of the mathematically most accessible examples of quantum many-body systems. In particular, they have recently received strong attention in attempts to understand the phenomenon of many-body localization (MBL) in disordered quantum systems. The central task here is to describe and prove the existence or absence of quantum transport in terms of the propagation of group waves through the system. These lectures will have two main objectives. First, we will introduce some of the main concepts available to describe MBL, including zero-velocity Lieb-Robinson bounds and area laws for quantum entanglement, and illustrate them in the example of the disordered XY chain. Then we will discuss recent work establishing MBL in the droplet phase of the disordered XXZ chain, a model where physicists expect a many-body localization transition. Along the way we will mention some of the many open problems which remain.

**Ancillary Talks:**

- Simon Becker

  **Title:** Magnetic Oscillations in Graphene

  **Abstract:** We consider the simplest model of graphene in a magnetic field given by a hexagonal quantum graph. Using semi-classical methods (with the strength of the magnetic field as the small parameter) we obtain a geometric description of the density of states showing asymmetry seen in physical experiments but not in commonly used perfect cone approximations. That density of states can then be used to see magnetic oscillations such as the de Haas-van Alphen effect. Joint work with M Zworski.

- Alex Bols

  **Title:** The Quantum Hall effect and edge modes in the Many-Body context

  **Abstract:** The exact quantization of the Hall conductance, discovered experimentally by von Klitzing in 1980, is by now well understood in the single particle context. A complete description of the effect however, must take into account interactions and therefore requires a many-body description. A proof of quantization of the Hall conductance of an interacting electron system was first achieved by Hastings and Michalakis. I will present a simpified account of this proof, and indicate how similar techniques can be used to prove the existence of gapless edge modes in the many-body context.

- Mahmoud DarAssi

  **Title:** Soret-driven convection in colloidal suspensions

  **Abstract:** The effect of sedimentation and thermophoresis on the threshold conditions for the onset of convection is represented by an experimental parameter, $\beta$, which is introduced by Chang et. al. (2008). The experimental setup showed that, $\beta$, is a function of the particle radius, $r_p$. The graph of $\beta(r_p)$ is approximately an inverted parabola with two zero crossings in the range $5\,nm \leq r_p \leq 125\,nm$. We conducted both linear and weakly nonlinear stability analyses for both dilute and moderately concentrated cases. The analysis focuses on the particle dominated convection regime, for which the onset is steady, and to disturbances having infinitely long wave-length. We put forth stability criteria as functions of relevant parameters and compare them with those obtained through the binary mixture formalism.

- Ryan Demuse

  **Title:** Bounding Mixing Time for Glauber Dynamics

  **Abstract:** Exponential random graph models are important models in the study of modern networks. These models are able to simulate common network tendencies through local graph features. We study the efficiency of single site Glauber dynamics for vertex weighted exponential random graphs. In particular, we study the lower bound on the mixing time for the Glauber dynamics in a specific region of the parameter space, the high temperature phase. We show using a coupon collecting argument and spectral methods that the mixing time in the high temperature phase is $\Omega(n \log n)$.

- Terry Easlick

  **Title:** An upper bound for mixing Glauber dynamics

  **Abstract:** Exponential Random Graphs models (ERGMs) give us a tool that we may use to study the characteristics of various networks; for instance, connectivity and reciprocity. Such networks tend to arise in economics, biology and sociology. We consider the mixing time for Glauber dynamics during the high temperature phase. We are able to utilize path coupling methods to establish an upper bound during the high temperature phasenamely, $O(n \log n)$.

- Christoph Fischbacher

  **Title:** Droplet States in Quantum $XXZ$ Spin Systems on General Graphs

  **Abstract:** We introduce $XXZ$ Quantum Spin Systems on general graphs. To this end, we will define the so-called $N$-th symmetric product of a graph and show the relation of the $XXZ$ Hamiltonian to a direct sum of Schrodinger operators on these $N$-th symmetric products. We will also discuss the existence of spectral gaps above the droplet band for quasi-one-dimensional graphs. This is joint work with G. Stolz.

- Rohan Ghanta

  **Title:** Pekar's Ansatz and the Ground-State Symmetry of a Bound Polaron

  **Abstract:** The Fröhlich polaron is a model of an electron interacting with the quantized optical modes (phonons) of an ionic crystal, and it has received considerable attention over the years as a toy model in quantum field theory. We shall consider a Fröhlich polaron bound in a symmetric Mexican hat-type potential. In the strong-coupling limit, the ground-state energy is given (to a leading order) by Pekar's minimization problem. Furthermore, for all values of the coupling parameter, the ground state is unique and therefore invariant under rotations. We show, however, that all the minimizers of the corresponding Pekar problem are nonradial. Moreover, assuming the nonradial minimizers are unique up to rotation, we prove in the strong-coupling limit that the radial electron density of the ground state converges in a weak sense to a rotational average of the densities of the minimizers.

- Benjamin Hinrichs

  **Title:** On the Non-Relativistic Limit of Quantum Electrodynamics

  **Abstract:** We consider the model of a massless bosonic photon field interacting with a fermionic Dirac field of mass $M$ in the charge $Q = 1$ sector. A non-relativistic limit of the Hamilton operator is studied. We discuss the rise of the one-particle Pauli-Fierz operator, known from non-relativistic QED, in this limit. The physical interpretation of arising error terms and the idea of proof will shortly be sketched in the end.

- Tori Hudgins

  **Title:** Approximating the Variance of Coefficients for the Characteristic Polynomial of Binary Quantum Graphs

  **Abstract:** Quantum graphs provide a simple model of quantum mechanics in systems with complex geometry. A quantum graph has an associated unitary quantum evolution operator. We study the coefficients of the characteristic polynomial of the quantum evolution operator for the family of binary graphs. The variance of the coefficients can be expressed as a sum over pairs of pseudo orbits, collections of periodic orbits. Expanding this sum diagrammatically, we approximate the variance by evaluating the first two terms in the expansion.

- Michal Jex

**Title:** Discrete spectrum of delta' interaction supported by non-closed manifolds

**Abstract:** In this talk we present a rigorous definition of delta' interaction supported by non-closed manifolds. We show that for the weak coupling the discrete spectrum disappears. This is surprising and previously unknown behavior for delta' interaction because the case of the attractive delta' interaction supported by a closed manifold always has at least one negative eigenvalue. For the two-dimensional case, we give a sufficient condition for the absence and existence of the discrete spectrum. The sufficient condition is dependent on the coupling constant and the curvature of the curve supporting the interaction.

- Markus Lange

**Title:** Renormalization Analysis of Second Order Split-up for Spin-Boson Models

**Abstract:** We consider a quantum mechanical system, which is modeled by a Hamiltonian acting on a finite dimensional space with degenerate eigenvalues coupled to a field of relativistic bosons. Provided a mild infrared assumption and a split up assumption holds, we show existence of the ground state and ground state eigenvalue of the interacting system using operator theoretic renormalization. Moreover we prove that the ground state eigenvalue is an analytic function of the coupling constant in a cone with apex at the origin.

- Marius Lemm

**Title:** Spectral gaps without frustration

**Abstract:** In spin systems, the existence of a spectral gap has far-reaching consequences. "Frustration-free" spin systems form a subclass that is special enough to make the spectral gap problem amenable and, at the same time, broad enough to include physically relevant examples. We discuss "finite-size criteria," which allow to bound the spectral gap of the infinite system by the spectral gap of finite subsystems.

- Ben Li

**Title:** Exact values of quantum violations in low-dimensional Bell correlation inequalities

**Abstract:** The famous Clauser-Horne-Shimony-Holt (CHSH) inequality certifies a quantum violation, by a factor $\sqrt{2}$, of correlations predicted by the classical view of the world in the simplest possible nontrivial measurement setup (two systems with two dichotomic measurements each). In such setting, this is the largest possible violation, which is known as the Tsirelson bound. In this talk we calculate the exact values of quantum violations for the other Bell correlation inequalities that appear in the setups involving up to four measurements; they are all smaller than $\sqrt{2}$. While various authors investigated these inequalities via numerical methods, our approach is analytic. We also include tables summarizing facial structure of Bell polytopes in low dimensions.

- Long Meng

  **Title:** Regularity of $N$ quantum bodies solutions and application to numerical analysis

  **Abstract:** The electronic quasi-relativistic equation is one kind of quasi-relativistic equation with the Coulomb forces in the field. It is more general than the electronic Schrödinger equation which has been analyzed by Harry Yserentant with the Pauli principle by Hardy inequality. In this talk, we will use the similar idea to analyze the electronic relativistic equation. But it is much more arduous. We can not get coercivity directly, and according to cusp analysis, we even can not use integer derivatives. In this article, we will overcome all of them by some ingenious skills. It is shown that these wave functions possess certain square integrable mixed weak derivatives of order $N/4 + 1/2$ with the $N$ the number of electrons, across the singularities of the interaction potentials.

- Alvin Moon

  **Title:** Spectral gaps in one dimensional quantum spin systems

  **Abstract:** In this talk, I will state the problem of spectral gap stability in the setting of one dimensional quantum spin systems. I will discuss an extension of quasi-local and spectral flow methods to frustration free systems with local topological quantum order and open boundary conditions. I will also demonstrate applications of the spectral flow in systems with fermion interactions.

- Bruno Nachtergaele

  **Title:** Stability of the superselection sectors of Kitaevs abelian quantum double models

  **Abstract:** Kitaevs quantum double models provide a rich class of examples of two-dimensional lattice systems with topological order in the ground states and a spectrum described by anyonic elementary excitations. The infinite volume ground states of the abelian quantum double models come in a number of equivalence classes called superselection sectors. We prove that the superselection structure remains unchanged under uniformly small perturbations of the quantum double Hamiltonians. (joint work with Matthew Cha and Pieter Naaijkens)

- Renaud Raquepas

  **Title:** The Landauer principle and repeated interaction systems

  **Abstract:** We will discuss Landauer's principle for repeated interaction systems consisting of a reference quantum system $S$ in contact with a environment $E$ consisting of a chain of independent quantum probes. The system $S$ interacts with each probe sequentially, for a given duration, and the Landauer principle relates the energy variation of $E$ and the decrease of entropy of $S$ via the entropy production of the dynamical process. We will consider refinements of the Landauer bound at the level of the full statistics associated to a two-time measurement protocol of the energy of $E$. The emphasis will be put on the adiabatic regime where the environment, consisting of a large number $T$ of probes, displays variations of order $1/T$ between the successive probes, and the measurements take place initially and after $T$ interactions.

- Alexei Rybkin

  **Title:** On the Hankel operator approach to the Cauchy problem for the KdV equation

  **Abstract:** As is well-known, many problems in the theory of completely integrable systems can be formulated in terms of Riemann-Hilbert boundary problems. This has been used (explicitly or implicitly) since the late 1980s. On the other hand, it is also well-known that the Riemann-Hilbert problem is closely related to the theory of Hankel and Toeplitz operators. Moreover, since the 1960s (and implicitly even earlier) the former has stimulated the latter. But, surprisingly enough, while having experienced a boom at the same time, soliton theory and the theory of Hankel and Toeplitz operators have not shown much of direct interaction.

  In the KdV context, we construct a Hankel operator which symbol is conveniently represented in terms of the scattering data for the Schrodinger operator associated with the initial data. Thus the spectral properties of this Schrodinger operator can be directly translated into the spectral properties of the Hankel operator. The latter then yield properties of the solutions to the KdV equation through explicit formulas. This allows us to recover and improve on many already known results as well as a variety of new ones. The main feature of this approach is that it applies to large classes of initial data far beyond the classical realm. For instance, we can handle low regularity initial data, lift any decay assumption at minus infinity, and significantly relax the decay at plus infinity. In this talk we discuss some representative results in this context focusing on well-posedness issues and basic properties of underlying solutions.

  Our approach is not restricted to the KdV. Moreover, we believe that the interplay between soliton theory and Hankel operators may be even more interesting and fruitful for some other integrable systems with richer than KdV structures.

- Axel Saenz

  **Title:** Geometric/Bernoulli Growth Process from Schur-Processes

  **Abstract:** This talk is based on a collaboration with Leo Petrov (UVa) and Alisa Knizel (Columbia). We introduce a discrete time and space TASEP model with mixed geometric and Bernoulli steps and probabilities given by Schur-symmetric functions. We use determinantal formulas to prove the limit shape for the model and find the Tracy-Widom distribution under the proper scaling. We are also able to extend these results (in distribution) to other models outside the scope of Schur processes by applying some recent result from Leo's work.

- Jeff Schenker
  **Title:** Localization, and resonant delocalization, of a disordered polaron
  **Abstract:** Polaron models describe the motion of a tracer particle interacting with a quantum field. The Holstein Hamiltonian describes a tight binding particle interacting with a field of harmonic oscillators. We will consider the Holstein Hamiltonian with the addition of on-site disorder in the tracer particle potential. Provided the hopping amplitude for the particle is small, we are able to prove localization for matrix elements of the resolvent, in particle position and in the field Fock space. These bounds imply a form of dynamical localization for the particle position that leaves open the possibility of resonant tunneling in Fock space between equivalent field configurations. Some related deformations of the Anderson model in which we can prove the existence of resonant tunneling will be presented, but the

exact nature of the dynamics for the disordered Holstein model remains open. (Joint work with Rajinder Mavi.)

- Oliver Siebert

  **Title:** Existence of ground states of translation-invariant Pauli-Fierz models

  **Abstract:** We consider translation-invariant Pauli-Fierz models describing an electron interacting with a quantized electromagnetic field. One can decompose the Hamiltonian with respect to the total momentum in a direct integral. For zero total momentum we show the existence of a ground state for all values of the coupling constant. In the case of non-zero momentum one has to pass to a coherent state representation inequivalent to the Fock representation in order that a ground state exists. Then we can prove the same result for almost all total momenta being small enough.

- Selim Sukhtaiev

  **Title:** A bound for the eigenvalue counting function for higher-order Krein Laplacians on arbitrary open sets

  **Abstract:** In this talk I will discuss a bound for the eigenvalue counting function (for strictly positive eigenvalues) for higher-order Krein Laplacians. The latter are particular self-adjoint extensions of minimally defined, positive integer powers of the Laplacian on arbitrary open, bounded sets. The bound extends to open, finite volume domains of finite width, subject to a compact Sobolev embedding property, and shows the correct high-energy power law behavior familiar from Weyl asymptotics. Based on joint work with M. Ashbaugh, F. Gesztesy, A. Laptev, and M. Mitrea.

- Hagop Tossounian

  **Title:** Thermostat's in Kac's Model, and the Partially Thermostated Kac Model

  **Abstract:** Mark Kac introduced a microscopic, $N$ particle, stochastic model in an attempt to study the Boltzmann equation. Through a device now known as propagation of chaos, he showed that his model is connected to the simpler Kac-Boltzmann equation. In this talk I will introduce Kac's model and thermostats in the context of Kac's model, and will introduce the effect on equilibration of thermostating only a fraction of the particles in the system.

- Arnaud Triay

  **Title:** Derivation of the Gross-Pitaevskii energy

  **Abstract:** We consider $N$ trapped bosons in $\mathbb{R}^3$ interacting via a pair potential $w$ which has a long range of dipolar type. We show the convergence of the energy and of the minimizers for the many-body problem towards those of the dipolar Gross-Pitaevskii functional, when $N$ tends to infinity. In addition to the usual cubic interaction term, the latter has the long range dipolar interaction. The results hold under the assumption that the two-particle interaction is scaled in the form $N^{3\beta-1}w(N^\beta x)$ for some $0 \leq \beta < \beta_{\max}$ with $\beta_{\max} = 1/3 + s/(45 + 42s)$ where $s$ is related to the growth of the trapping potential.

- Anna Vershynina

  **Title:** Data Processing Inequality and the stability of the recovery map

  **Abstract:** The Data Processing Inequality (DPI) states that the Umegaki relative entropy is non-increasing under the action of completely positive trace preserving (CPTP) maps. A theorem of Petz says that there is equality in DPI if and only if both states can be recovered perfectly after passing through a CPTP map. Such recovery map is called Petz recovery map. A standing problem is to obtain a proper lower bound on the difference between relative entropies of input and output states. We provide a quantitative version of Petz's theorem, where the lower bound contains a distance between a state and its Petz's recovered state. The novelty of the result is that for the first time the distance measure contains the original Petz recovery map. Moreover, I will present stability bounds for the quasi-relative entropies defined in terms of an operator monotone decreasing functions, which also includes the distance measure of the state and its Petz's recovered state. The present treatment is developed in the context of finite dimensional von Neumann algebras where the results are already non-trivial and of interest in quantum information theory. (Joint work with Eric A. Carlen)

- Mei Yin

  **Title:** Ground states for exponential random graphs

  **Abstract:** We propose a perturbative method to estimate the normalization constant in exponential random graph models as the weighting parameters approach infinity. As an application, we give evidence of discontinuity in natural parametrization along the critical directions of the edge-triangle model. Joint work with Rajinder Mavi.