Soret-driven convection in colloidal suspensions

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Colloidal Suspensions

- Colloidal suspension is a mixture of a base fluid, such as water, and small size particles, less than one micron in diameter.
- Examples of colloidal suspensions are: wall paint, Italian salad dressing, muddy water and even ice cream.
- The process of natural convection in colloidal suspensions has been investigated by the research community since the experimental discovery of Choi (2001). That is the addition of a small volume fraction of particles can enhance the thermal conductivity of the fluid.

Colloidal Suspensions

- Convection in colloidal suspensions is theoretically investigated using the same mathematical formalism as convection in binary mixtures. However, convection in colloidal suspensions is characterized by

- Large size of the particle ($1nm$ - $1\mu\, m$) implies longer diffusion time scales.

- Much smaller Lewis numbers $\tau \approx 10^{-4}$ compared to $\tau \approx 10^{-2}$.

- Stronger Soret effect leading to much larger separation ratios.

- Drastic reduction in the threshold for convection onset implies more difficult convection experiments to determine the threshold conditions.

Particulate Medium

- Particulate medium has been used to study the onset of Rayleigh-Bénard convection in colloidal suspension.
- Particulate medium model takes into account thermophoresis, sedimentation and Brownian diffusion effects.
- The importance of using particulate medium lies in the ease at which experimental and analytical results can be compared.

Particulate Medium

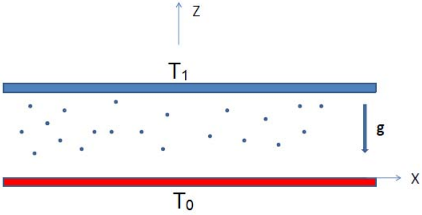


Figure 1: Sketch of a dilute colloidal suspension of solid particles confined between two plates maintained at temperatures T_0 and $T_1 < T_0$, $\mathbf{g}$ is the gravitational vector. The thickness of the suspension layer is H.

Particulate Medium

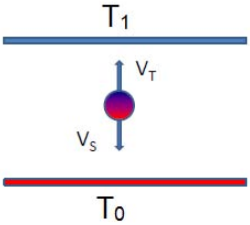


Figure 2: A suspended particle under the effects of both thermophoresis upward and sedimentation downward.

Experimental work of Chang, Mills and Hernandez

- The graph of $\beta = \frac{6\,\pi\,\mu\,D_T\,\Delta\,T}{T\,k_B}\,r_p - \frac{4\,(\rho-1)\,g\,\pi\,\mu\,H}{3\,\nu\,k_B\,T}\,(r_p)^3$ is an approximately inverted parabola with two zero crossings.

- The first zero crossing corresponding to the molecular size particles that is assumed in the binary mixture models.

- The other zero crossing corresponds to larger size particles, although still in the nanosize range, and represents the case of nearly balanced effects of sedimentation and thermophoresis.

Experimental work of Chang, Mills and Hernandez

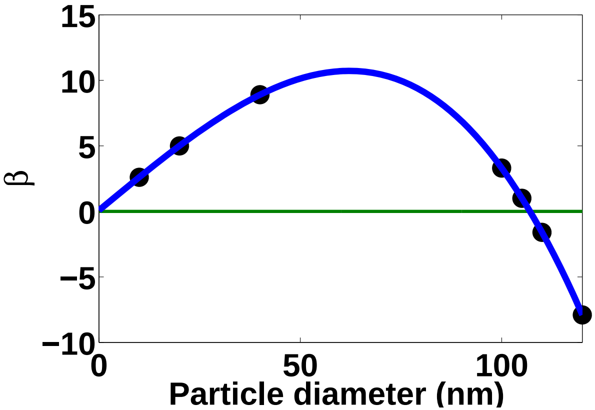


Figure 3: The particle speed β as a function of the particle's diameter.

Mathematical Model

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\mathrm{Pr}^{-1}\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) = -\nabla p + \nabla\cdot(F\nabla\mathbf{u}) + (R\,T - B\,C)\mathbf{k} \tag{2}$$

$$\frac{\partial T}{\partial t} + \frac{d\,T_B}{d\,z}\mathbf{u}\cdot\mathbf{k} + \mathbf{u}\cdot\nabla T = \nabla^2 T \tag{3}$$

$$\tau^{-1}\left(\frac{\partial C}{\partial t} + \frac{d\,C_B}{d\,z}\mathbf{u}\cdot\mathbf{k} + \mathbf{u}\cdot\nabla C\right) = \nabla^2 C - \beta\frac{d\,C}{d\,z} + \beta_T\left[(C + C_B)\,\nabla^2 T + \frac{d\,C_B}{d\,z}\,\frac{d\,T}{d\,z} + \nabla C\cdot\nabla T\right] \tag{4}$$

Where$\mathrm{Pr} = \nu/\kappa$ the Prandtl number, $\tau = D/\kappa$ the Lewis number, $S = \gamma_C\, C_M/(\gamma_T\,\Delta\, T)$ is the separation ratio, C_M is the mean particle volume fraction, $R = \gamma_T\,\Delta\, T\, g\, h^3\, C_M^2/\nu\,\kappa,$ *and* $B = R\,S$ are the thermal and solutal Rayleigh numbers, respectively.

Linear eigenvalue problem

$$F(C) = 1 + 2.5C_B, \qquad D = \frac{d}{dz}, \quad S_T = \frac{\Delta T\, D_T}{D_0}$$

where $F(C)$ represents the dependence of viscosity on particle concentration.

$$F(C)\nabla^4 W + 2\, DF(C)\, \nabla^2(DW) + D^2 F(C)\,(D^2 W) = (R + \Psi)\, {\nabla_H}^2 C,$$

$$\nabla^2 \phi = \tau^1 (DC_B(z) W + \phi_t) - \beta\, D\phi - (a\, S_T\, DC_B(z))\, \phi - a\, S_T\, C_B(z)\, D\phi$$

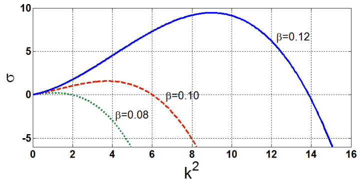
$$W = DW = 0, D\phi = \beta\, \phi, \quad z = 0, 1$$

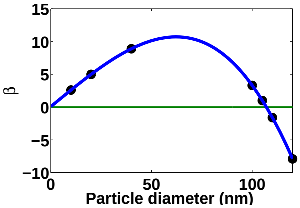
where W is the vertical component of the velocity and Ψ is the contribution from the thermophoretic force in the momentum equation,

$$\Psi = \frac{9\, \phi_m\, \Delta T\, H^2}{2 r_p^2\, \mathcal{D}_0}$$

Dispersion relation

As $\beta \to 0$ steady convection sets in with an infinite wavelength. This convection state corresponds to two physical situations.





Stability Diagram

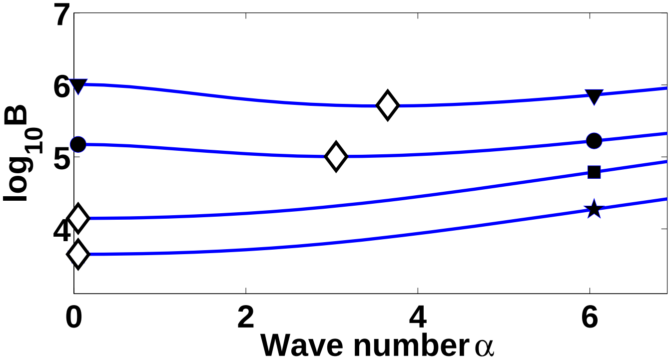


Figure 4: Plot of the buoyancy number $log_{10}B$ as function of the wavenumber α for the case $m = 2.5$ and for a selected set of values of β, $\beta = 9\,(\bigtriangledown)$, $7\,(\circ)$, $1\,(\Box)$ and $\beta = 0.2\,(\star)$. The diamond symbol $(\diamondsuit)$ depicts the location of the minimum value of B.

Threshold conditions for dilute suspensions-molecular size particles

$$R_c(0,a) = \frac{720\,\tau\,(1 - a\,\Phi_m)}{S\,\beta\,[1 + \tau\,(1 - a\,\Phi_m)]} + \frac{\Psi}{S}$$

where,
$\Psi = (9/2)\,\beta_T\,(H/r_p)^2\,\Phi_m$, due to the thermophoretic force term. We retrieve the expression for R_c that is derived within the binary mixture formalism, $R_c = 720\,\tau/S$, if $a = 0$ and ignore the thermophoretic force term.

Threshold conditions for dilute suspensions-nano size particles

$$R_c = \frac{720\,\tau\,(1 + m\,\Phi_m)\,(1 - a\,\Phi_m)}{S\,[1 + a\,\Phi_m(2 - (2\,\beta_T + \beta_S)/\beta) + (1 - a\,\Phi_m(2\,\beta_T + \beta_S)/2\,\beta))\tau\,(\beta_T/\beta)]}$$

For $\Phi_m \ll 1$, it reduces to

$$R_c = \frac{720\,\tau}{S}\left(\frac{1 - 4.05\,\Phi_m}{1 + \tau\,(\beta_T/\beta)}\right)$$

where the factor between parentheses reflects the modification of R_c by the dual effects of particle diffusion and sedimentation.

Region of validity of the asymptotic analysis

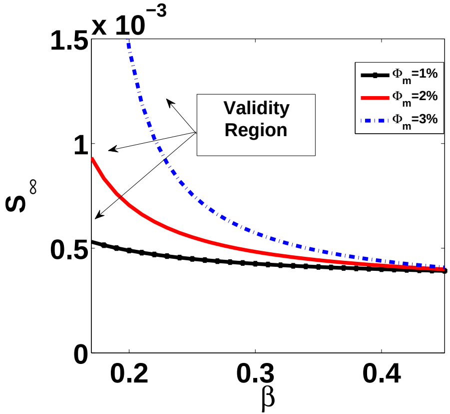


Figure 5: Plot of the lower bound $\mathcal{S}_\infty$ as a function of $\widehat{\beta}$ for $\Phi_m = 1\%$ (connected dots) line, $\Phi_m = 2\%$ (solid) line and $\Phi_m = 3\%$ (dashed) line.

Plot of the threshold values as function of β

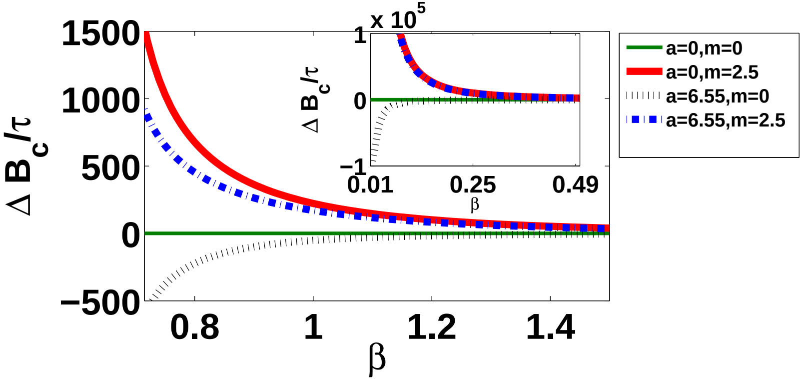


Figure 6: Plot of $\Delta B_C/\tau = (B_c(0,0) - B_c(m,a))/\tau$ as function of β for the case, $\beta_S \neq 0$, $\tau = 10^{-4}$, $\Phi_m = 2\%$.

Summary

- We Investigated the onset of Rayleigh-Bénard convection in a suspension of inert solid particles.
- We adopted a particulate medium model. This model is more consistent with experiments (better connection between model and physical parameters).
- For $0 < \beta \ll 1$, which experimentally corresponds to two distinct particle radii, the convection onset is to disturbances having infinitely long wavelength.

Summary

- A small wavenumber asymptotic analysis yields threshold conditions as function of the experimental parameters, such as the height of the fluid cell and particle radius. The findings are in the form analytical expression which help in interpreting the role of the various experimental and physical parameters. In particular, we quantified the role of sedimentation.
- A small wave-number expansion yields threshold conditions as function of the experimental parameters, such as the height of the fluid cell and the particle radius.

$$R_c(0,a) \sim \frac{720\,\tau\,(1-a\Phi_m)}{\mathcal{S}\,[1+\tau\,(1-a\Phi_m)]} + \frac{6\,\pi\,(H^2/LW)N\,(r_p/H)\,\beta_T^2}{\mathcal{S}}$$

Thank You