Math 651: Winter 2006
Integrable Systems, Nonlinear Waves, and Soliton Theory
Suggestions of Papers for Presentations

The list below contains several papers on topics of relevance to our course. In consulation with me if you wish, please choose a paper from this list (or find another relevant paper — this list is by no means exhaustive). Each of you will have the opportunity to give a 35-minute mini-lecture to our class on the subject of the paper you choose to read. We will have these mini-lectures on Thursday, April 6 (2 presentations), Tuesday, April 11 (2 presentations), Thursday, April 13 (2 presentations), and Tuesday, April 18 (1 presentation). Note: if you choose a paper that we have talked about in class, you should be prepared to present to the class aspects of the paper that were not already discussed.

Don't be afraid to select a lengthy paper, or another textbook on the subject: you can concentrate on just a small part of the paper/book for your presentation.

Classic papers.

- The first use of the word "soliton": N. J. Zabusky and M. D. Kruskal, *Phys. Rev. Lett.*, **15**, pp. 240–243, 1965.
- The inverse-scattering solution of KdV: C. S. Gardner, J. M. Greene, M. D. Kruskal, and R. M. Miura, *Phys. Rev. Lett.*, **19**, pp. 1095–?, 1967.
- Lax equations, and the first suggestion of the possibility of there being more remarkable equations like KdV: P. D. Lax, "Integrals of nonlinear equations of evolution and solitary waves", *Comm. Pure Appl. Math.*, **21**, pp. 467–490, 1968.
- The discovery of the integrability of the nonlinear Schrödinger equation: V. E. Zakharov and A. B. Shabat, "Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media", *Sov. Phys. JETP*, **34**, pp. 62–69, 1972.
- One of the earliest general schemes for finding and solving integrable equations: M. J. Ablowitz, D. J. Kaup, A. C. Newell, and H. Segur, "The inverse-scattering transform — Fourier analysis for nonlinear problems", *Stud. Appl. Math.*, **53**, pp. 249–315, 1974.

Specific applications.

- Solitons in nonlinear fiber optics: A. Hasegawa and F. Tappert, "Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers, I; anomalous dispersion", *Appl. Phys. Lett.*, **23**, pp. 142–144, 1973. See also the paper by L. Mollenauer in the collection indicated below.
- Water waves. Here there are many, many papers. The integrable equations that describe water waves in various limits are many: Korteweg-de Vries equation, Boussinesq equation, Camassa-Holm equation, Kadomtsev-Petviashvili equation, nonlinear Schrödinger equation, Davey-Stewartson equation, Benjamin-Ono equation, intermediate long-wave equation, *etc.* A paper on any of these equations would be appropriate. Also, consider the following provocatively-titled paper: A. I. Dyachenko and V. E. Zakharov, "Is free-surface hydrodynamics an integrable system?", *Phys. Lett. A*, **190**, pp. 144–148, 1994.
- Plasma physics. See, perhaps, E. I. Schulman, "On the integrability of equations of short and long wave resonance interaction", *Sov. Phys. Dokl.*, **26**, pp. 691–692, 1981. See also the references therein.
- An integrable equation arising in the theory of general relativity is the Ernst equation. As a start, see C. Klein and O. Richter, "Riemann-Hilbert problems for the Ernst equation and fibre bundles", *J. Geom. Phys.*, **30**, pp. 331–342, 1999. References therein may also be useful.

- Applications of the sine-Gordon equation:
  - The sine-Gordon equation in classical differential geometry: L. P. Eisenhart, *A treatise on the differential geometry of curves and surfaces*, Dover, New York, 1960 (see p. 318).
  - The sine-Gordon equation and the theory of superconducting Josephson junctions: A. Barone, F. Esposito, C. J. Magee, and A. C. Scott, "Theory and applications of the sine-Gordon equation", *Riv. Nuovo Cim.*, **1**, pp. 227–267, 1971.

Mathematical aspects.

- Scattering and inverse-scattering theory.
  - P. Deift and E. Trubowitz, "Inverse scattering on the line", *Comm. Pure Appl. Math.*, **32**, pp. 121–251, 1979.
  - R. Beals and R. R. Coifman, "Scattering and inverse scattering for first order systems", *Comm. Pure Appl. Math.*, **37**, pp. 39–90, 1984.
- Symmetries. Hamiltonian aspects. See the expository paper by F. Magri, "A short introduction to Hamiltonian PDEs", (Fifth Workshop on Partial Differential Equations, Rio de Janeiro, 1997), *Mat. Contemp.*, **15**, pp. 213–230, 1998.
- Algebraic geometry.
  - An excellent review article: B. A. Dubrovin, "Theta functions and non-linear equations", *Russ. Math. Surveys*, **36**, pp. 11–92, 1981.
  - Another excellent review article: I. M. Krichever, "Methods of algebraic geometry in the theory of non-linear equations", *Russ. Math. Surveys*, **32**, pp. 185–213, 1977.
  - The paper that started the interaction between soliton theory and algebraic geometry: B. A. Dubrovin, V. B. Matveev, and S. P. Novikov, "Non-linear equations of Korteweg-de Vries type, finite-zone linear operators, and Abelian varieties", *Russ. Math. Surveys*, **31**, pp. 59–146, 1976.
  - Schottky problem and the Novikov conjecture. The Schottky problem asks which symmetric positive-definite matrices are period matrices of Riemann surfaces? Novikov conjectured that the answer was those whose theta functions satisfy the Kadomtsev-Petviashvili (KP) equation of water wave theory. This was proved by Shiota: T. Shiota, "Characterization of Jacobian varieties in terms of soliton equations", *Invent. Math.*, **83**, pp. 333–382, 1986. A review article on this topic: B. Dubrovin, "Dispersion relations for nonlinear waves and the Schottky problem", in *Important developments in soliton theory* (A. Fokas and V. E. Zakharov, eds.), pp. 86–98, Springer, Berlin, 1993.
- Asymptotics for integrable nonlinear waves.
  - H. Flaschka, M. G. Forest, and D. W. McLaughlin, "Multiphase averaging and the inverse spectral solution of the Korteweg-de Vries equation", *Comm. Pure Appl. Math.*, **33**, pp. 739–784, 1980.
  - An excellent review article for modulation theory: B. A. Dubrovin and S. P. Novikov, "Hydrodynamics of soliton lattices", *Sov. Sci. Rev. C Math. Phys.*, **9**, pp. 1–136, 1993.
  - P. Deift and X. Zhou, "A steepest descent method for oscillatory Riemann-Hilbert problems. Asymptotics for the mKdV equation", *Ann. Math.*, **137**, pp. 295–368, 1993.
  - P. Deift and X. Zhou, "Long-time asymptotics for integrable systems. Higher order theory", *Comm. Math. Phys.*, **165**, pp. 175–191, 1994.

- P. D. Lax and C. D. Levermore, “The small dispersion limit of the Korteweg-de Vries equation, I, II, and III”, *Comm. Pure Appl. Math.*, **36**, pp. 253–290 (part I), 571–593 (part II), and 809–830 (part III), 1983.
- A review article: P. D. Lax, C. D. Levermore, and S. Venakides, “The generation and propagation of oscillations in dispersive initial-value problems and their limiting behavior”, in *Important Developments in Soliton Theory* (A. S. Fokas and V. E. Zakharov, eds.), pp. 205–241, Springer Verlag, 1992. (Note, there are many excellent review articles in this book.)

- Orthogonal polynomials.
  - P. Deift, T. Kriecherbauer, K. T.-R. McLaughlin, S. Venakides, and X. Zhou, “Asymptotics for polynomials orthogonal with respect to varying exponential weights”, *Internat. Math. Res. Notices*, **16**, pp. 759–782, 1997.
  - P. Deift, T. Kriecherbauer, K. T.-R. McLaughlin, S. Venakides, and X. Zhou, “Strong asymptotics of orthogonal polynomials with respect to exponential weights”, *Comm. Pure Appl. Math.*, **52**, pp. 1491–1552, 1999.
  - P. Deift, T. Kriecherbauer, K. T.-R. McLaughlin, S. Venakides, and X. Zhou, “Uniform asymptotics for polynomials orthogonal with respect to varying exponential weights and applications to universality questions in random matrix theory”, *Comm. Pure Appl. Math.*, **52**, pp. 1335–1425, 1999.
- Random matrix theory.
  - P. A. Deift, A. R. Its, and X. Zhou, “A Riemann-Hilbert approach to asymptotic problems arising in the theory of random matrix models, and also in the threory of integrable statistical mechanics”, *Ann. Math.*, **146**, pp. 149–235, 1997.
- Hirota theory and τ-functions. For an exposition by the inventor, see R. Hirota, “Direct method of finding exact solutions of nonlinear evolution equations”, in *Bäcklund transformations, the inverse scattering method, solitons, and their applications (Workshop Contact Transformations, Vanderbilt University, Nashville, Tenn., 1974)*, pp. 40–68, Lecture Notes in Math., **515**, Springer, Berlin, 1976.

Other integrable systems.

- Vector nonlinear Schrödinger equations (polarized pulses in optical fibers): S. V. Manakov, “On the theory of two-dimensional stationary self-focusing of electromagnetic waves”, *Sov. Phys. JETP*, **38**, pp. 248–253, 1974.
- Toda lattice. Many interesting papers.
- Ablowitz-Ladik equations. M. J. Ablowitz and J. F. Ladik, “A nonlinear difference scheme and inverse scattering”, *Stud. Appl. Math.*, **55**, pp. 213–229, 1976. Also, see M. J. Ablowitz and J. F. Ladik, “Nonlinear differential-difference equations and Fourier analysis”, *J. Math. Phys.*, **17**, pp. 1011–1018, 1976.
- Intermediate long-wave equation.
- Benjamin-Ono equation.
- Davey-Stewartson equation (a two-dimensional version of NLS).
- Camassa-Holm equation.

- KP equation (a two-dimensional version of KdV). See, for example, S. V. Manakov, "The inverse scattering transform for the time-dependent Schrödinger equation and Kadomtsev-Petviashvili equation", *Physica D*, **3**, pp. 420–427, 1981.

- Boussinesq equation.

- Maxwell-Bloch system. M. Agrotis, N. M. Ercolani, S. A. Glasgow, and J. V. Moloney, "Complete integrability of the reduced Maxwell-Bloch equations with permanent dipole", *Physica D*, **138**, pp. 134–162, 2000.

- 3-wave interaction system. See the text by Ablowitz and Clarkson, or E. V. Doktorov, "Spectral transform and solitons for the three-wave coupling model with nontrivial boundary conditions", *J. Math. Phys.*, **38**, pp. 1–13, 1997.

Survey papers.

- A wonderful collection of papers: M. Atiyah, J. D. Gibbon, and G. Wilson (eds.) "New Developments in the Theory and Application of Solitons", *Phil. Trans. R. Soc. Lond. A*, **315**, 1985. Contents include:
  - J. D. Gibbon, "A survey of the origins and physical importance of soliton equations".
  - J. B. Keller, "Soliton generation and nonlinear wave propagation".
  - P. van Moerbeke, "Algebraic geometrical methods in Hamiltonian mechanics".
  - G. Wilson, "Infinite-dimensional Lie groups and algebraic geometry in soliton theory".
  - N. M. Ercolani and H. Flaschka, "The geometry of the Hill equation and of the Neumann system".
  - A. C. Scott, "Davydov solitons in polypeptides".
  - L. F. Mollenauer, "Solitons in optical fibers and the soliton laser".
  - R. S. Ward, "Integrable and solvable systems, and relations among them".
  - M. Atiyah and N. J. Hitchin, "Low-energy scattering of non-Abelian magnetic monopoles".

Textbooks.

- T. Miwa, M. Jimbo, and E. Date, *Solitons. Differential equations, symmetries and infinite-dimensional algebras*, Cambridge Tracts in Mathematics, **135**, Cambridge University Press, Cambridge, 2000.

- M. J. Ablowitz and P. A. Clarkson, *Solitons, nonlinear evolution equations and inverse scattering*, London Mathematical Society Lecture Note Series, **149**, Cambridge University Press, Cambridge, 1991.

- L. D. Faddeev and L. A. Takhtajan, *Hamiltonian methods in the theory of solitons*, Springer, Berlin, 1987.

- A. C. Newell, *Solitons in mathematics and physics*, CBMS-NSF Regional Conference Series in Applied Mathematics, **48**, SIAM, Philadelphia, 1985.

- M. J. Ablowitz and H. Segur, *Solitons and the inverse scattering transform*, SIAM Studies in Applied Mathematics, **4**, SIAM, Philadelphia, 1981.