Math 129 – Exam 2 – Spring 2011SOLUTIONS –

(1a) The Right Approximation with $n = 2$.

(1b) The correct statements were

$$\text{LEFT}(N) < \int_{-15}^{11} \arctan(x)dx < \text{RIGHT}(N)$$

$$\text{MID}(N) < \int_{-15}^{0} \arctan(x)dx < \text{TRAP}(N)$$

$$\text{TRAP}(N) < \int_{0}^{11} \arctan(x)dx < \text{MID}(N)$$

(1c) The right Riemann sum is

$$\sum_{i=1}^{n} f(x_i)\Delta x$$

(2a) Since the integrand $\frac{x^5}{e^{-x}+x}$ does not go to zero as $x \to +\infty$, the integral diverges.

(2b) It is an improper integral because at $x = 0$ the denominator vanishes. The value of the integral is 2001.

(2c) The integral diverges: behaves like $1/\sqrt{x}$, so $p = 1/2$.

(2d) Since $0 \leq \sin^6(x) \leq 1$, we know that $0 \leq \frac{\sin^6(x)}{(1+x)^2} \leq \frac{1}{(1+x)^2}$, and that implies that $\int_0^L \frac{\sin^6(x)}{(1+x)^2}dx \leq \int_0^L \frac{1}{(1+x)^2}dx$. When you evaluate the last integral and let $L \to \infty$, it converges to 1. Thus C is a correct answer. A cannot be correct, since the integral is certainly greater than 0, and B cannot be correct since the integral converges.

(3) One must build a sum starting from the surface area of the *floor* of the ground floor, and ending with the floor of the 10th floor. That yields the value $232,500$ cubic feet. Then one builds a second sum, starting with the surface area of the *ceiling* of the ground floor, and ending with the surface area of the ceiling of the 10th floor (which is the surface area of the roof). That yields the value $205,500$ cubic feet. The average of these two is $219,000$ cubic feet.

(4a) The area of the cross section is obtained from an isosceles right triangle whose longest side has length e^x, the area of this triangle is $\frac{1}{4}e^{2x}$. The chunk volume is $\frac{1}{4}e^{2x}\Delta x$, and the volume is $\int_0^1 \frac{1}{4}e^{2x}dx$.

(4b) If H is the chunk height measured from the bottom, then volume of the chunk is

$$\pi\left(\frac{100-H}{2}\right)^2 \Delta H \ .$$

The force needed to counteract gravity is $2\pi\left(\frac{100-H}{2}\right)^2 \Delta H$ pounds. The distance this chunk travels is H feet, and so the work required for this chunk is $2\pi H\left(\frac{100-H}{2}\right)^2 \Delta H$. The total work is

$$\int_0^{100} 2\pi H\left(\frac{100-H}{2}\right)^2 dH = \frac{\pi}{2}\int_0^{100} H(100-H)^2 dH \ .$$

If instead one lets h denote the distance of the chunk *from the top*, then the final answer is

$$\frac{\pi}{2}\int_0^{100} (100-h)h^2 dh \ .$$

(5a) The only answer that is correct is$\sum_{i=0}^{n-1} \rho(x_i)\Delta x$.

(5b) Chopping the city up into concentric rings, each ring thickness being Δr, we consider a chunk at radius r from the center. This chunk has area which you can get by computing the area of the outer circle and subtracting the are of the inner circle, as we did in class, or the area can be obtained by figuring out the circumference of the chunk: $2\pi r$, and multiplying by the chunk thickness Δr. Either way, the approximation to the chunk area is

$$\text{Chunk Area} \approx 2\pi r\Delta r \ .$$

Then the population *within that chunk* is $(5000(5-r))(2\pi r\Delta r)$. Thus the total population is

$$\int_1^3 (5000(5-r))(2\pi r)dr \approx 356000.$$