Name (please print): ________________________________

- Show all algebraic work to receive as much credit as possible.
- Please turn OFF all cell phones and stow them out of sight. All textbooks, notes, etc. must also be put away. Keep your eyes on your own paper.

1.) The sign function, denoted \( \text{sgn} \), is defined by

\[
\text{sgn}(x) = \begin{cases} 
-1 & x < 0 \\
0 & x = 0 \\
1 & x > 0
\end{cases}
\]

Determine the following limits, if they exist. If they do not, explain why.

(i) \( \lim_{x \to 0^+} \text{sgn}(x) \)  
(ii) \( \lim_{x \to 0^-} \text{sgn}(x) \)  
(iii) \( \lim_{x \to 0} \text{sgn}(x) \)  
(iv) \( \lim_{x \to 0} |\text{sgn}(x)| \)

2.) Use that Sandwich Theorem to determine \( \lim_{x \to 0} x^2 \cos(1/x) \).

3.) Determine the following limits

(i) \( \lim_{x \to -2} \frac{x^2 + x - 2}{x^2 - 2x - 3} \)  
(ii) \( \lim_{x \to \infty} \frac{x^3 - x^4 + 3x}{x^5 - x^4 + 12} \)  
(iii) \( \lim_{x \to -\infty} \frac{x^2 - x^4 + x^7}{x^7 + x^2 - 2} \)  
(iv) \( \lim_{x \to \infty} \frac{x^2 + 2x^4}{x + 7x^3 + 1} \)

4.) Suppose \( f \) and \( g \) are continuous functions such that \( g(2) = 6 \) and

\[
\lim_{x \to 2} [3f(x) + f(x)g(x)] = 36.
\]

Determine \( f(2) \).
5.) Determine any removable discontinuities of,

\[ f(x) = \frac{x^2 - x - 6}{x - 3}. \]

If any removable discontinuities are found, how would you redefine \( f \) at these points in order to make \( f \) continuous?

6.) Below is the graph of \( f, f', \) and \( f'' \).

Identify \( f, f', \) and \( f'' \).

7.) Determine the derivatives of the following functions

(i) \( f(x) = 2(x^4 - x)^3 + 5 \)

(ii) \( h(x) = \frac{\csc(x)}{2x + 5} \)

(iii) \( g(x) = \log_3(x^2) \cdot 5^{11x} \)

(iv) \( q(x) = e^{\cot(5x)} \)

8.) Determine the equation of the line tangent to the graph of \( g(t) = 5t/(1 + t^2) \) at the point \((2, 2)\).
9.) Below is the graph of \( f \).

\[ y \]

\[ x \]

Determine the values of \( x \) at which \( f \) is not differentiable, and explain why \( f \) is not differentiable at those points.

10.) Consider the function

\[ f(x) = x \ln(x^2) \]

Determine all local and absolute extrema. Be sure to indicate whether they are minimums or maximums.

11.) If \( f(2) = 20 \) and \( f'(x) \geq 3 \) for \( 2 \leq x \leq 5 \), how small can \( f(5) \) be?

12.) Below is the graph of \( f' \):

\[ y = f'(x) \]

Use this graph to determine the intervals in which \( f \) is concave down and concave up, and the \( x \)-values of any inflection points.
13.) A baseball team plays in a stadium that seats 55,000 people. When the tickets were priced at $10 the average attendance per game is 27,000. The team has determined that for each dollar that the ticket price is lowered the average attendance per game will increase by 3,000. What should ticket prices be in order to maximize revenue?

14.) A Norman window has the shape of a rectangle surmounted by a semicircle:

![Window Diagram]

If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is let in.

15.) Determine the following indefinite integrals

(i) \( \int \frac{1}{2} (\csc^2(x) - \csc(x) \cot(x)) \, dx \)  
(ii) \( \int \frac{-1}{t\sqrt{25t^2 - 1}} \, dt \)

(iii) \( \int \left( \frac{1}{x} - \frac{5}{x^2 + 1} \right) \, dx \)  
(iv) \( \int \sin(10x) + \frac{3}{\sqrt{1 - 25t^2}} \, dt \)

16.) Suppose \( g'''(x) = \cos(x) \) with \( g(0) = 1, g'(0) = 2, \) and \( g''(0) = 3. \) Determine \( g. \)
17.) Let \( F(x) = \int_{2}^{x} f(t) \, dt \) where \( f \) is the function whose graph is given below:

![Graph of \( y = f(t) \)]

Which of the following values is the largest?

- (a) \( F(0) \)
- (b) \( F(1) \)
- (c) \( F(2) \)
- (d) \( F(3) \)
- (e) \( F(4) \)

18.) If \( \int_{0}^{9} f(x) \, dx = 37 \) and \( \int_{0}^{9} g(x) \, dx = 16 \) determine

\[
\int_{0}^{9} [2f(x) + 3g(x)] \, dx
\]

19.) Determine upper and lower bounds for the definite integral \( \int_{1}^{e} \ln(x) \, dx \).

20.) Determine the derivative of the following

\[
f(x) = \int_{1-3x}^{1} \frac{u^3}{1 + u^2} \, du
\]

21.) Determine the following integrals

- (i) \( \int_{0}^{\pi/4} \sec(\theta) \tan(\theta) \, d\theta \)  
- (ii) \( \int_{0}^{4} (4 - t) \sqrt{t} \, dt \)  
- (iii) \( \int_{-5}^{4} \pi \, dv \)
22.) Determine the following integrals

(i) \( \int \frac{(\ln(x))^2}{x} \, dx \)  
(ii) \( \int_{0}^{a} x\sqrt{a^2 - x^2} \, dx \)  
(iii) \( \int e^x \sqrt{1 + e^x} \, dx \)

23.) Determine the area of the shaded region below