TEACHING STATEMENT

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1. Philosophy

During the spring of 2008, I taught an introductory linear algebra course at the University of Arizona. One of the students in my class was a marine–whom I will call Mark–who had made the unusual choice to pursue a major in mathematics. Based on his earlier classes, Mark thought that the study of mathematics consisted in learning to execute procedures quickly and accurately. Moreover, much of his training in the Marines was of a similarly algorithmic nature. The linear algebra course, however, while primarily a computational course, does emphasize proofs to some extent. Mark was an intelligent and dedicated student, but he felt lost when trying to construct proofs. Without a procedure to follow, he did not know how to begin.

When Mark came to my office for help and told me about his background, I realized that he needed a procedural description of how one constructs a proof, even if such a procedure could not be complete. I therefore gave him the following sequence of steps:

1. Write down all of the assumptions given in the problem as formally as possible–ideally in equations–at the top of a piece of paper.
2. Write down the statement you are trying to prove in a similarly formal way at the bottom of the piece of paper.
3. Using established principles, operations, or theorems, construct a path from the statements at the top to the statement at the bottom.

Although this prescription is not really much more substantive than the algorithm proposed by Feynman–write down the problem, think very hard, write down the solution–it was very helpful for Mark. When viewed as an explicit procedure with distinct steps, the first two of which are essentially mechanical, the task of constructing a proof seems much more approachable to someone who is accustomed to following procedures. To make the third step seem less intimidating, I explained that, at least in undergraduate classes, the necessary components can typically be found in the section of the textbook to which the given problem is associated. In other words, the third step is akin to putting together a jigsaw puzzle: the required pieces are known and available; it only remains to assemble them.

Making the transition from procedural to abstract mathematics is a daunting step for some students. Easing them through this transition has been one of the most rewarding aspects of teaching for me. The series of steps described above has now become my standard advice for students who are struggling with proofs. I am currently running problem sessions for an introduction to proofs class, and I have used this description repeatedly to help students get started on proofs. In some cases, no further input from me is required.

Even in lower level classes where proofs are not emphasized, I try to indicate that mathematics is about ideas, not rote computations. When teaching calculus, I stressed over and over that, although learning to calculating derivatives by hand is part of the purpose of the course, the primary objective is to learn what derivatives mean and how they can be applied to real-world problems. Moreover, most calculations can be carried out much more quickly and accurately by a computer than by a human. When teaching Gaussian elimination in linear algebra, I explained that, since computers can carry out this procedure in a fraction of a second, the exam would not consist in a competition to see who could row-reduce a 6x6 matrix the fastest, as it might in some linear algebra courses.

It is easy to become bewildered by the profusion of minutiae that are necessary in mathematics. The technical definitions required to make proofs rigorous often obscure the intuitive notions which they represent, and algorithmic processes are often simpler to understand and remember when broken into large groups of steps. For each concept that I present as a teacher, in addition to giving a detailed description and complete examples, I try to provide a broad overview. For people who are not detail-oriented, an overview is helpful because it can bring coherence to a potentially intimidating mass of steps and notation. This is particularly true in courses such as vector calculus, in which students are exposed to a large amount of new and potentially confusing notation.

On the other hand, people who are more detail or procedure-oriented are sometimes inclined toward a brute force approach–i.e., rote memorization–to learning mathematics. They may be comfortable with lengthy procedures involving complicated notation, and they will try to memorize these procedures one step at a time. This may be effective in high school mathematics classes, but in college-level classes, too much material is covered during the course of a semester to memorize. For these people, an overview encourages a broader perspective and allows them to distinguish each procedure from the rest.

Even individual constructions may be too complicated to master without somehow minimizing the number of individual points one needs to remember. At the beginning of the linear algebra course, I asked the students to memorize a couple of simple proofs. The idea was to introduce them to proofs via a familiar mode of thinking. What I found, however, was that a sizable portion of the class struggled with this seemingly straightforward task. After several students came to my office for help, it became clear that they were trying to memorize proofs by rote, almost symbol by symbol in some cases. I explained that learning a proof is much like learning a joke. One memorizes only the main points and the overall scheme into which they fit. The details can be filled in as needed. As they had discovered, trying to memorize all of the details can be impractical or impossible, and can obscure the overall picture.

Ideally, almost no memorization is necessary in mathematics. Many mathematical concepts and definitions are motivated by either physical phenomena or geometric considerations. As a geometer, visualization is the primary tool that I use to keep track of abstract concepts, and I try to encourage this approach in my students. My lectures include many pictures, either drawn by hand or produced using software. For example, for vector calculus, I used Maple to draw the gradient of a function of three variables along with one of its level sets. The advantage of using Maple was that I could grab and rotate this image with the mouse, effectively producing an animation. The goal, of course, was to illustrate the fact that the gradient of a function is normal to its level sets, something that can be difficult to see in a static image (in the three variable case).

I have also used software to aid in visualization, both in lecture and in interactive labs. One lab that worked particularly well employed a Java applet for visualizing 3-dimensional vector fields (available at http://www.falstad.com/vector3d/). The lab (available at www.math.arizona.edu/ mlamb/math223/fall2007/vectorfieldlab.pdf), asked students to find formulas for vector fields that produce specified types of particle motion. During lectures, I have used software to produce surfaces in 3-dimensions, cross-products of vectors, images of linear maps, and various other images that are difficult to draw by hand.

For many students, the greatest challenge in learning mathematics does not lie in visualizing objects in 3-dimensions or in adopting a more abstract mode of thinking, but simply in overcoming their fear of the subject. Some people are intimidated by mathematics to the point that their brains effectively cease to function when taking an exam or when asked a question in class.

One of the most important responsibilities for a teacher of mathematics is to convince students that they can, in fact, handle the material, and that they can succeed in the class. I have always tried to create a respectful, open, inclusive atmosphere in the classroom. I actively encourage questions, and I do my best to make any question or comment seem worthwhile. Over the past few years, I have worked on addressing the issue of intimidation more directly. In particular, I now make a point of laying out in an explicit way exactly what students need to do in order to succeed in the class. My hope is that this leads students to believe that success is possible. I also do the same for tests, emphasizing that if they have read the textbook and done the homework, completing the tests will be a straightforward process. This is a difficult problem, and my attempts to solve it have not been entirely successful, but I think I do as well as most teachers in this regard.

I work very hard to present the beauty and power of mathematics in a form that is accessible and engaging. To the extent possible, I try to accommodate different learning styles, spending many hours helping students outside of class if necessary. I emphasize understanding over computation and memorization. I try to to ease the transition from procedural to abstract mathematics by motivating mathematical formalism with pictures and concrete examples. I do my best to create a classroom atmosphere in which students are comfortable and engaged. There is no doubt that I still have a great deal to learn about teaching, but I am working hard to improve, and I am enjoying the process. I hope to continue teaching mathematics throughout my professional career.

2. Experience

2.1. **Undergraduate Instruction.** As a graduate student at the University of Arizona, I have taught a wide range of undergraduate courses, including

- College Algebra (3 times)
- Precalculus
- Calculus I (twice)
- Calculus II (twice)
- Vector Calculus
- Linear Algebra.

For all of the courses listed above, I was fully responsible for giving lectures, designing homework assignments, writing tests, and grading. The College Algebra, Calculus I, Calculus II, and Vector Calculus courses were coordinated in the sense that all sections shared common syllabi and final exams. For the Linear Algebra course, I had considerable freedom in what I chose to cover, and I wrote the final exam.

During the Fall 2008 semester, I served as the graduate T.A. for an introduction to proofs class. My responsibilities included running problem sessions and holding office hours.

2.2. **Graduate Instruction.** Beginning in 2003, the mathematics department at the University of Arizona has held a rigorous, five-day workshop each August for the incoming graduate students. Each workshop consisted of a series of lectures delivered by faculty and problem sessions run by senior graduate students. The purpose of these workshops is to review undergraduate mathematics from a "graduate" perspective, as well as to introduce new students to some of the members of the department. In 2006 and 2008, I served as a senior student mentor, which involved helping students with problems and answering general questions about graduate school.

I also wrote a project on multilinear algebra for the workshop (available at www.math.arizona.edu/ mlamb/multilinearproject.pdf), beginning with tensor products and wedge products of vector spaces and culminating in an interpretation of the div, grad, and curl operators from vector calculus as special cases of the d-operator from differential geometry. My intention was to give the incoming students a head start in the differential geometry course, which is typically the most challenging of the first year core courses.

This project was in fact a reformulation of a set of notes I had written on the subject as part of my VIGRE fellowship during the summer of 2007 (available at www.math.arizona.edu/ mlamb/tensors.pdf). The aim of these notes was to supply, via concrete examples and informal descriptions, some practical ways of thinking about differential forms and multivector fields. In particular, I tried to supply explanations that were missing from most of the standard textbooks.

3. Goals

In the future, I hope to teach a wide variety of courses, including real and complex analysis, abstract algebra, topology, and statistics. I am particularly excited about teaching an introduction to proofs class, if the possibility arises.

I also plan to make further use of technology as a teaching aid, particularly by creating interactive labs using Maple, Matlab, and other software. I am interested in writing new software for this purpose. In particular, I have a number of ideas for Java applets which illustrate connections between formal definitions and geometric interpretations in calculus. Although I have a fair amount of programming experience, these sorts of projects would be ideal for undergraduates.

Another project I would like to supervise is an iterated "chicken" tournament. The game of chicken (two cars drive towards each other, whoever swerves first loses) is a variation on the prisoner's dilemma in which the order of the outcomes has been changed. In an iterated tournament, contestants submit algorithms for playing an opponent in chicken multiple times. These algorithms are then encoded in a computer program, and a round-robin tournament is conducted. The algorithm that accumulates the most points wins. One can also run genetic simulations to determine which algorithms are most effective in the long run.

Finally, I plan to work on actively engaging students during class. I have experimented with labs and with assigning small groups of students to work on problems together, but most of my teaching has followed a traditional lecture format. In the future, I will incorporate more group work and labs into my classes. I will also ask students to present solutions on the board and perhaps base a percentage of their grades on participation. Ultimately, I would like to encourage–and enforce, to some extent–participation from every student in the class, especially from those who lack confidence in their abilities.