Hand the answers to these questions in at the start of class. Make sure your pages are stapled together if you have more than one and that your name and the date/number of the homework are written on the front page. Show all work, work down the page and use only one column. Give all answers in exact form (if appropriate give an approximation also).

1. The picture below shows a plot in the $xy$ plane of the polar curves $r = \theta$ and $r = 2\theta$ for $0 \leq \theta \leq 2\pi$.

![Plot of polar curves](image)

Find the area of the region bounded by the two curves and $\theta = 2\pi$.

2. (This is the example I mucked up in class).
   (a) Write the integral below in spherical coordinates:
   \[
   \int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{2-x^2-y^2}}^{\sqrt{x^2+y^2}} \, xy \, dz \, dy \, dx.
   \]
   Hint: Find where $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{2 - x^2 - y^2}$ intersect. Then draw a picture of the solid region you are integrating over.
   (b) (2 bonus points) Evaluate the integral.
   (c) (2 more bonus points) Write the integral in cylindrical coordinates, like we did in class, and evaluate it that way. Check that you get the same answer.

3. Consider the object bounded by the $z = 0$ plane and the surface $z = 9 - x^2 - y^2$. Find a number $a$ such that the plane $z = a$ slices the object into two pieces of equal volume.
4. The center of mass of a 2-dimensional object of density $\rho(x, y)$ and mass $M$ covering a region $R$ in the plane is $(\bar{x}, \bar{y})$ where

$$\bar{x} = \frac{1}{M} \int_R x\rho(x, y) \, dA$$
$$\bar{y} = \frac{1}{M} \int_R y\rho(x, y) \, dA.$$ 

Let $R$ be the L shaped region pictured below with the origin at the bottom left corner and a constant density $\rho(x, y) = 1$.

(a) Show that its center of mass is given by

$$\bar{x} = \bar{y} = \frac{b^2 + ab - a^2}{2(2b - a)}.$$ 

(b) Find one example of a pair $(a, b)$ so that the center of mass is outside the object (some kind of educated guess and check is probably easier than solving inequalities).