Student’s Name __________________________________________________________

By signing my name below, I agree that I am following all rules and regulations set forth by the Code of Academic Integrity. Furthermore, I agree that I am following all rules set by my instructor and by the course policy for this exam. This includes ensuring that all calculator programs except possibly EVALUATE and QUADRATIC FORMULA have been deleted.

Signature: ___________________________ Date: ___________________________

Section I – Multiple Choice

Circle the correct answer. No partial credit will be given.

1. Find the second digit to the right of the decimal point in the number \( \log_{\frac{3}{5}}(8) \).
   
   (A) 4      (B) 5      (C) 6
   
   (D) 7      (E) 8

2. For \( x, y, z > 0 \) the expression \( \log_2 \left( \frac{xy}{\sqrt{z}} \right) \) is equal to

   (A) \( \frac{(\log_2 x)(\log_2 y)}{\sqrt{\log_2 z}} \)      (B) \( \log_2 x + \log_2 y - \sqrt{\log_2 z} \)
   
   (C) \( (\log_2 x)(\log_2 y) - \frac{1}{2} \log_2 z \)      (D) \( \frac{\log_2 x + \log_2 y}{\sqrt{\log_2 z}} \)
   
   (E) \( \log_2 x + \log_2 y - \frac{1}{2} \log_2 z \)

3. How much MORE will you earn in an account that compounds continuously than in an account that compounds interest quarterly if you invest $2000 for 8 years at an interest rate of 12% per year?

   (A) $73.23      (B) $271.47      (C) $36,639.75
   
   (D) $55.24      (E) $89.83
4. Consider the function \( f(x) = C \cdot b^x \) where \( C > 0 \) and \( 0 < b < 1 \). Determine which of the following statements are true.

(i) The range of \( f \) is all real numbers.
(ii) The graph of \( f \) has a horizontal asymptote at \( y = C \).
(iii) The function \( f \) is decreasing everywhere.
(iv) The graph of \( f \) has no \( x \)-intercepts.

(A) i, ii and iii only   (B) iii and iv only   (C) ii, iii and iv only

(D) i, ii and iv only   (E) All of the above

Section II – Short Answer

Answer in the space provided. Show your work.

1. Solve the equation

\[
\frac{1}{2} - \log_{16}(x - 3) = \log_{16} x
\]

exactly.
2. In the 1930s Cane Toads were introduced to Australia\(^1\). Since then they have multiplied and spread to many parts of northern and eastern Australia. If \( A \) is the area in thousands of square kilometres that has been colonized by Cane Toads and \( t \) is the number of years since 1939 then we can model the area colonized by \( A = Pe^{rt} \).

(a) If the area colonized by toads was 44 thousand square kilometres in 1939 and 61 thousand square kilometres in 1945 find \( P \) and \( r \).

(b) The area of mainland Australia is 7 690 thousand square kilometres. When does our model predict the toads will reach all parts of the mainland?

\(^{1}\text{They were introduced to control the Cane Beetle. Unfortunately they couldn’t jump high enough to eat them from off the cane stalks...}\)
3. Consider the function \( f(x) = \ln(x + 1) + 2 \).

(a) Find the \( x \) and \( y \) intercepts of the graph of \( y = f(x) \).

(b) Graph \( y = f(x) \).

(c) Explain why \( f(x) \) is a one-to-one function. Find the inverse function \( f^{-1}(x) \).
4. Solve the equation $4^x = 10^{2x+1}$ exactly.

5. The Richter magnitude, $M$, of an earthquake is defined by the formula

$$M = \log \left( \frac{A}{A_0} \right)$$

where $A$ is the amplitude of the earthquake and $A_0$ is a constant. Solve this equation to give $A$ as a function of $M$. 
6. The amount of a radioactive isotope remaining after \( t \) years is given by

\[ A = Ce^{-0.002t}. \]

After how many years will there be 70% of the original amount of the isotope remaining?

7. A function \( y = ae^{bx} \) passes through the points \((-6, 12)\) and \((-3, 3)\). Find \( a \) and \( b \).