Test 3 Review

MATH 110 · Section 2 · Spring 2008       Instructor: Martin Leslie

Formulas (that will be given on the test)

\[ A = Pe^{rt} \quad A = P \left(1 + \frac{r}{n}\right)^{nt} \]

Things you should know

3.3 Rational Functions
(a) For a reduced rational function, how to find the x and y intercepts, vertical asymptotes, horizontal/slant asymptotes and put it all together in a graph.
(b) How to graph a non-reduced rational function.
(c) How to work with models of the form \( at/(t + b) \)

4.1 Exponential functions
(a) The basic shape of the graph \( y = b^x \) and thus transformations like \( y = C \cdot b^x \).
(b) The discrete compound interest model \( A = P \left(1 + \frac{r}{n}\right)^{nt} \).

4.2 The Natural Exponential
(a) The basic shape of the graph \( y = e^x \) and transformations of it.
(b) The continuous compound interest model \( A = Pe^{rt} \).

4.3 Inverse Functions
(a) What it means for \( f \) and \( g \) to be inverses: \( f(g(x)) = x \) and \( g(f(x)) = x \). Graphically they are reflections through the line \( y = x \). The domain of \( f \) must be the range of \( g \) and the range of \( g \) must be the domain of \( f \).
(b) A function is one-to-one if every output comes from only one input. Algebraically this means \( f(x_1) = f(x_2) \) implies \( x_1 = x_2 \). Graphically this is the horizontal line test.
(c) A function has an inverse if and only it is one-to-one. To find the inverse of a function \( f(x) \) given by a formula set \( y = f^{-1}(x) \) so \( f(y) = x \). Then solve for \( y \). For a function given by a table switch the inputs and outputs. For a function given graphically reflect through \( y = x \).
(d) What inverse means: it’s really just the reverse of \( f \). If \( f(a) = b \) then \( f^{-1}(b) = a \). In particular \( f(f^{-1}(x)) = x \) and \( f^{-1}(f(x)) = x \).

4.4 Logarithms
(a) The definition \( \log_b N = x \) where \( b^x = N \). This should convince you of some basic identities like \( \log_b 1 = 0 \) and \( \log_b b = 1 \).
(b) The function \( \log_b x \) is the inverse function of \( b^x \). This gives us more identities \( b^{\log_b x} = x \) and \( \log_b(b^x) = x \).
(c) The domain, range and graph of \( y = \log_b x \).
(d) Common and natural logs: \( \log x = \log_{10} x \) and \( \ln x = \log_e x \).
(e) The log laws and how to use them to condense/expand expressions
   (i) \( \log_b(PQ) = \log_b P + \log_b Q \)
   (ii) \( \log_b(P/Q) = \log_b P - \log_b Q \)
   (iii) \( \log_b(P^r) = r \log_b P \).

4.5 Logarithmic and Exponential Equations
(a) Change of base formula
\[ \log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a} = \frac{\ln x}{\ln a} \]
(b) Exponential equations.
(c) Logarithmic equations. (Solutions may be extraneous so check your answers).
(d) Simple applications (given the equation).
Some practice questions

3.3 Rational Functions
(a) Find the $x$ and $y$ intercepts, all asymptotes and then graph the reduced rational functions
\[ f(x) = \frac{5 - 2x}{x - 2}, \quad g(x) = \frac{x - 3}{x + 2}^2 \quad \text{and} \quad h(x) = \frac{3x^2 - 2x - 1}{3x - 5}. \] (This is three separate questions).
(b) Graph the non–reduced function \( f(x) = \frac{x^3 - x^2}{4x - 4} \).
(c) Textbook Section 3.3 question 49 (page 206).

4.1 Exponential functions
(a) Describe the shape of the graph of \( y = C \cdot b^x \). You should have four different cases depending on what \( C \) and \( b \) are.
(b) Find the amount of money you will have after 10 years, having deposited $200 in an account at 8% interest compounded twice a year.

4.2 The Natural Exponential
(a) Graph the function \( y = e^{x+2} - 15 \). What are its domain, range, \( x \) and \( y \) intercepts and end behavior?
(b) Find the amount of money you will have after 10 years, having deposited $200 in an account at 8% interest compounded continuously. Compare this to the amount above where it was compounded twice a year.

4.3 Inverse Functions
(a) Draw a function that has an inverse. Graph that inverse. Draw a function that does not have an inverse.
(b) Check that \( f(x) = 1/(x - 5) \) is one–to–one both algebraically and graphically. Find \( f^{-1} \).

4.4 Logarithms
(a) Expand \( \log(xy^2) \).
(b) Condense \( \log_2(x) - (1/3) \log_2(y) \).
(c) Find the domain of \( f(x) = \log(x + 2) \). Graph \( y = f(x) \).

4.5 Logarithmic and Exponential Equations
(a) Calculate \( \log_2 9 \).
(b) Solve \( \log_3 x = 4 \).
(c) Solve \( (3/5)^x = 25/9 \).
(d) Solve \( 2000 = 540e^{0.06t} \).
(e) Solve \( \ln x = 2x - 3 \ln(1/x) \).
(f) Solve \( (1/2)^6 \cdot 2^x = 2^x \).
(g) Solve \( \log(x^2 - 1) - \log(x + 4) = \log(x) \).
(h) The population of an endangered insect species, in millions, is given by
\[
P(t) = 100 \left( \frac{1}{2} \right)^{t/10}
\]
where \( t \) is the number of years after the year 2000. In what year will the insect population first fall below 20 million?

Other review
(1) Workbook: review questions 103-183 starting page 218
(2) Other pages of the workbook that look relevant.
(3) Textbook exercises/chapter test.