Test 3 Review

MATH 110 · Section 2 · Spring 2008 Instructor: Martin Leslie

Formulas (that will be given on the test)

$$A = Pe^{rt} \qquad A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Things you should know

3.3 Rational Functions
- (a) For a reduced rational function, how to find the x and y intercepts, vertical asymptotes, horizontal/slant asymptotes and put it all together in a graph.
- (b) How to graph a non–reduced rational function.
- (c) How to work with models of the form $at/(t+b)$

4.1 Exponential functions
- (a) The basic shape of the graph $y = b^x$ and thus transformations like $y = C \cdot b^x$.
- (b) The discrete compound interest model $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

4.2 The Natural Exponential
- (a) The basic shape of the graph $y = e^x$ and transformations of it.
- (b) The continuous compound interest model $A = Pe^{rt}$.

4.3 Inverse Functions
- (a) What it means for f and g to be inverses: $f(g(x)) = x$ and $g(f(x)) = x$. Graphically they are reflections through the line $y = x$. The domain of f must be the range of g and the range of g must be the domain of f.
- (b) A function is one–to–one if every output comes from only one input. Algebraically this means $f(x_1) = f(x_2)$ implies $x_1 = x_2$. Graphically this is the horizontal line test.
- (c) A function has an inverse if and only it is one–to–one. To find the inverse of a function $f(x)$ given by a formula set $y = f^{-1}(x)$ so $f(y) = x$. Then solve for y. For a function given by a table switch the inputs and outputs. For a function given graphically reflect through $y = x$.
- (d) What inverse means: it's really just the reverse of f. If $f(a) = b$ then $f^{-1}(b) = a$. In particular $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

4.4 Logarithms
- (a) The definition $\log_b N = x$ where $b^x = N$. This should convince you of some basic identities like $\log_b 1 = 0$ and $\log_b b = 1$.
- (b) The function $\log_b x$ is the inverse function of b^x. This gives us more identities $b^{\log_b x} = x$ and $\log_b(b^x) = x$.
- (c) The domain, range and graph of $y = \log_b x$.
- (d) Common and natural logs: $\log x = \log_{10} x$ and $\ln x = \log_e x$.
- (e) The log laws and how to use them to condense/expand expressions
  - (i) $\log_b(PQ) = \log_b P + \log_b Q$
  - (ii) $\log_b(P/Q) = \log_b P - \log_b Q$
  - (iii) $\log_b(P^r) = r \log P$.

4.5 Logarithmic and Exponential Equations
- (a) Change of base formula

$$\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}.$$

- (b) Exponential equations.
- (c) Logarithmic equations. (Solutions may be extraneous so check your answers).
- (d) Simple applications (given the equation).

Some practice questions

3.3 Rational Functions

(a) Find the x and y intercepts, all asymptotes and then graph the reduced rational functions $f(x) = (5-2x)/(x-2)$, $g(x) = (x-3)/(x+2)^2$ and $h(x) = (3x^2-2x-1)/(3x-5)$. (This is three separate questions).

(b) Graph the non–reduced function $f(x) = (x^3-x^2)/(4x-4)$.

(c) Textbook Section 3.3 question 49 (page 206).

4.1 Exponential functions

(a) Describe the shape of the graph of $y = C\cdot b^x$. You should have four different cases depending on what C and b are.

(b) Find the amount of money you will have after 10 years, having deposited \$200 in an account at 8% interest compounded twice a year.

4.2 The Natural Exponential

(a) Graph the function $y = e^{x+2} - 15$. What are its domain, range, x and y intercepts and end behavior?

(b) Find the amount of money you will have after 10 years, having deposited \$200 in an account at 8% interest compounded continuously. Compare this to the amount above where it was compounded twice a year.

4.3 Inverse Functions

(a) Draw a function that has an inverse. Graph that inverse. Draw a function that does not have an inverse.

(b) Check that $f(x) = 1/(x-5)$ is one–to–one both algebraically and graphically. Find f^{-1}.

4.4 Logarithms

(a) Expand $\log(xy^2)$.

(b) Condense $\log_2(x) - (1/3)\log_2(y)$.

(c) Find the domain of $f(x) = \log(x+2)$. Graph $y = f(x)$.

4.5 Logarithmic and Exponential Equations

(a) Calculate $\log_2 9$.

(b) Solve $\log_3 x = 4$.

(c) Solve $(3/5)^x = 25/9$.

(d) Solve $2000 = 540e^{0.06t}$.

(e) Solve $\ln x = 2x - 3\ln(1/x)$.

(f) Solve $(1/2)6^x = 2^x$.

(g) Solve $\log(x^2-1) - \log(x+4) = \log(x)$.

(h) The population of an endangered insect species, in millions, is given by

$$P(t) = 100\left(\frac{1}{2}\right)^{t/10}$$

where t is the number of years after the year 2000. In what year will the insect population first fall below 20 milion?

Other review

(1) Workbook: review questions 103-183 starting page 218

(2) Other pages of the workbook that look relevant.

(3) Textbook exercises/chapter test.