Appendix B.5

2. $\frac{25-x^2}{x-5} = \frac{(5-x)(5+x)}{x-5} = \frac{-(x-5)(x+5)}{x-5} = -(x+5)$

4. $\frac{x^2-x-20}{2x^2+7x-4} = \frac{(x-5)(x+4)}{(2x-1)(x+4)} = \frac{x-5}{2x-1}$

6. $\frac{a+b}{ax^2+bx^2} = \frac{a+b}{x^2(a+b)} = \frac{1}{x^2}$

8. $\frac{a^3b^2-27b^5}{(ab-3b^2)^2} = \frac{b^2(a^3-27b^3)}{(ab-3b^2)^2}$ difference of cubes

10. $\frac{(x-y)^2(a+b)}{(x^2-y^2)(a^2+2ab+b^2)} = \frac{(x-y)^{\cancel{2}}(a+b)}{(\cancel{x-y})(x+y)(a+b)^{\cancel{2}}} = \frac{x-y}{(x+y)(a+b)}$

14. $\frac{ax+3}{2a+1} \div \frac{a^2x^2+3ax}{4a^2-1} = \frac{\cancel{ax+3}}{\cancel{2a+1}} \times \frac{(2a-1)(2a+1)}{ax(\cancel{ax+3})}$

$= \frac{2a-1}{ax}$

18. $\frac{a^2-a-42}{a^4+216a} \div \frac{a^2-49}{a^3-6a^2+36a} = \frac{(\cancel{a-7})(a+6)}{\cancel{a}(a^3+216)} \cdot \frac{\cancel{a}(a^2-6a+36)}{(\cancel{a-7})(a+7)}$

$= \frac{(a^2-6a+36)(a+6)}{(a+6)(a^2-6a+36)(a+7)} = \frac{1}{a+7}$

20. $\frac{1}{3x} + \frac{1}{5x^2} - \frac{1}{30x^3} = \frac{1}{3x}\left(\frac{10x^2}{10x^2}\right) + \frac{1}{5x^2}\left(\frac{6x}{6x}\right) - \frac{1}{30x^3}$

$= \frac{10x^2}{30x^3} + \frac{6x}{30x^3} - \frac{1}{30x^3} = \frac{10x^2+6x-1}{30x^3}$

22. $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{bc}{abc}+\frac{ac}{abc}+\frac{ab}{abc}=\frac{bc+ac+ab}{abc}$

26. $1+\frac{1}{x}-\frac{1}{x^2}=\frac{x^2}{x^2}+\frac{x^2}{x}-\frac{1}{x^2}=\frac{x^2+x^2-1}{x^2}=\frac{2x^2-1}{x^2}$

27. $\frac{a}{x-1}+\frac{2ax}{(x-1)^2}+\frac{3ax^2}{(x-1)^3}=\frac{a}{x-1}\left(\frac{x-1}{x-1}\right)^2+\frac{2ax}{(x-1)^2}\cdot\frac{x-1}{x-1}+\frac{3ax^2}{(x-1)^3}$

$=\frac{a(x^2-2x+1)+2ax^2-2ax+3ax^2}{(x-1)^3}=\frac{ax^2\ 2ax+a+2ax^2-2ax+3ax^2}{(x-1)^3}$

$=\frac{6ax^2-4ax+a}{(x-1)^3}$

30. $\frac{x}{x+a}+\frac{a}{a-x}=\frac{x}{x+a}-\frac{a}{x-a}=\frac{x}{x+a}\cdot\frac{x-a}{x-a}-\frac{a}{x-a}\cdot\frac{x+a}{x+a}$

$=\frac{x^2-ax-ax-a^2}{x^2-a^2}=\frac{x^2-2ax+a^2}{(x-a)(x+a)}=\frac{(x-a)^2}{(x-a)(x+a)}=\frac{x-a}{x+a}$

32. $\frac{3}{2x+2}-\frac{5}{x^2-1}+\frac{1}{x+1}=\frac{3}{2(x+1)}-\frac{5}{(x-1)(x+1)}+\frac{1}{x+1}=$

$\frac{3(x-1)}{2(x-1)(x+1)}-\frac{10}{2(x-1)(x+1)}+\frac{x-1}{(x-1)(x+1)}=\frac{3x-3-10+x-1}{2(x-1)(x+1)}=\frac{4x-14}{2(x^2-1)}$

38. $\frac{\frac{4}{a}-a\left(\frac{a}{a}\right)}{\frac{2}{a}+1\left(\frac{a}{a}\right)}=\frac{\frac{4-a^2}{a}}{\frac{2+a}{a}}=\frac{(2+a)(2-a)}{a}\cdot\frac{a}{2+a}=2-a$

OR

$\frac{\frac{4}{a}-a}{\frac{2}{a}+1}\left(\frac{a}{a}\right)=\frac{4-a^2}{2+a}=\frac{(2-a)(2+a)}{2+a}=(2-a)$

↑ least common denominator for entire expression is "a"

40. $\dfrac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a}-\frac{1}{b}}\left(\dfrac{ab}{ab}\right) = \dfrac{b+a}{b-a}$

42. $\dfrac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}+\frac{1}{y}}\dfrac{x^2y^2}{x^2y^2} = \dfrac{y^2-x^2}{xy^2+x^2y} = \dfrac{(y-x)(y+x)}{xy(y+x)} = \dfrac{y-x}{xy}$

43. $\dfrac{\frac{1}{2+h}-\frac{1}{2}}{h} = \dfrac{\frac{1}{2+h}\cdot\frac{2}{2}-\frac{1}{2}\cdot\frac{2+h}{2+h}}{h} = \dfrac{\frac{2-2-h}{2(2+h)}}{h} = \dfrac{-h}{2(2+h)}\cdot\dfrac{1}{h}$

$= -\dfrac{1}{2(2+h)}$

44. $\dfrac{\frac{3}{x^2+h}-\frac{3}{x^2}}{h} = \dfrac{\frac{3}{x^2+h}\cdot\frac{x^2}{x^2}-\frac{3}{x^2}\cdot\frac{x^2+h}{x^2+h}}{h} = \dfrac{\frac{3x^2-3x^2-3h}{x^2(x^2+h)}}{h}$

$= \dfrac{-3h}{x^2(x^2+h)}\cdot\dfrac{1}{h} = \dfrac{-3}{x^2(x^2+h)}$

46. $\dfrac{x+\frac{xy}{y-x}}{\frac{y^2}{x^2-y^2}+1} = \dfrac{x-\frac{xy}{x-y}}{\frac{y^2}{(x-y)(x+y)}+1}\cdot\dfrac{(x-y)(x+y)}{(x-y)(x+y)} = \dfrac{x(x^2-y^2)-xy(x+y)}{y^2+x^2-y^2}$

$= \dfrac{x[x^2-y^2-xy-y^2]}{x^2} = \dfrac{x^2-xy}{x} = x-y$

49. $\left(\dfrac{1}{a^{-1}}+\dfrac{1}{a^{-2}}\right)^{-1} = (a+a^2)^{-1} = \dfrac{1}{a(1+a)}$

50. $(a^{-1}+a^{-2})^{-1} = \left(\dfrac{1}{a}+\dfrac{1}{a^2}\right)^{-1} = \left(\dfrac{a+1}{a^2}\right)^{-1} = \dfrac{a^2}{a+1}$

53. $$\frac{\frac{a+b}{a-b}+\frac{a-b}{a+b}}{\frac{a-b}{a+b}-\frac{a+b}{a-b}} \cdot \frac{ab^3-a^3b}{a^2+b^2}$$

$$= \frac{\frac{(a+b)^2+(a-b)^2}{a^2-b^2}}{\frac{(a-b)^2-(a+b)^2}{a^2-b^2}} \cdot \frac{ab(b^2-a^2)}{a^2+b^2}$$

$$= \frac{a^2+2ab+b^2+a^2-2ab+b^2}{\cancel{a^2-b^2}} \cdot \frac{\cancel{a^2-b^2}}{a^2-2ab+b^2-a^2-2ab-b^2} \cdot \frac{ab(b^2-a^2)}{a^2+b^2}$$

$$= \frac{2a^2+2b^2}{-4ab} \cdot \frac{ab(b^2-a^2)}{a^2+b^2} = \frac{\cancel{2}(a^2+b^2)\,ab(b^2-a^2)}{-\cancel{4}ab(a^2+b^2)} = \frac{a^2-b^2}{2}$$

55. $$\frac{\left(a+\frac{1}{b}\right)^a\left(a-\frac{1}{b}\right)^b}{\left(b+\frac{1}{a}\right)^a\left(b-\frac{1}{a}\right)^b} = \frac{\left(\frac{a+b}{b}\right)^a\left(\frac{a-b}{b}\right)^b}{\left(\frac{b+a}{a}\right)^a\left(\frac{b-a}{a}\right)^b}$$

$$= \left(\frac{\frac{a+b}{b}}{\frac{b+a}{a}}\right)^a\left(\frac{\frac{a-b}{b}}{\frac{b-a}{a}}\right)^b = \left(\frac{a+b}{b}\cdot\frac{a}{b+a}\right)^a\left(\frac{a-b}{b}\cdot\frac{a}{b-a}\right)^b$$

$$= \left(\frac{a}{b}\right)^a\left(\frac{a-b}{b}\cdot\frac{-a}{a-b}\right)^b = \left(\frac{a}{b}\right)^a\left(\frac{-a}{b}\right)^b$$