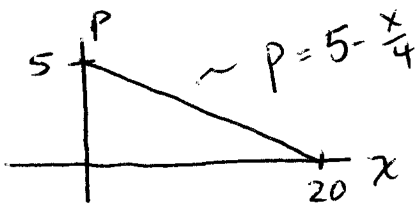
Section 4.4 Solutions - Setting up Equations

4.a Let A be the area of the shaded portion. We need the area of A as a function of x. So $A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}xy$. Since $P(x,y)$ lies on curve $y=\sqrt{x}$ this means $A(x) = \frac{1}{2}x(\sqrt{x}) = \frac{1}{2}x x^{1/2} = \frac{1}{2}x^{3/2}$. Domain is $(0,\infty)$

b. If P is perimeter and h is length of hypotenuse, by Pythagorean Theorem $x^2+y^2=h^2$ so $h=\sqrt{x^2+y^2}$ and since $y=\sqrt{x}$ $h=\sqrt{x^2+x}$. $P=x+y+h$ so $P(x) = x+\sqrt{x}+\sqrt{x^2+x}$ Domain is $(0,\infty)$

6. a. If h is the height, r the radius and V the volume and $V=\pi r^2 h$. It is given that $V = 12\pi \text{ in}^3$ so $12\pi = \pi r^2 h \Rightarrow h = \frac{12\pi}{\pi r^2} = \frac{12}{r^2}$ so $h(r) = \frac{12}{r^2}$ D: $(0,\infty)$

b. If S is surface area, the $S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r\left(\frac{12}{r^2}\right) = 2\pi r^2 + \frac{24\pi}{r}$ so $S(r) = 2\pi r^2 + \frac{24\pi}{r}$
D: $(0,\infty)$

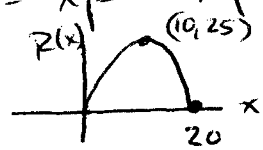
9. a. 

domain is $[0,20]$

b. $p=3$
$3 = 5 - \frac{x}{4} \Rightarrow -2 = -\frac{x}{4} \Rightarrow$
$x = 8$
So 8 units can be sold when unit price is \$3

c. $p = 5 - \frac{12}{4} = 5-3 = 2$
So to sell 12 units, price must be \$2.

d. Revenue = price × quantity sold $\Rightarrow R = xp = x\left(5-\frac{x}{4}\right)$
$= 5x - \frac{x^2}{4} \Rightarrow R(x) = 5x - \frac{x^2}{4}$ D $[0,20]$



e. $R(2) = \$9$ $R(14) = \$21$ $R(8) = \$24$
these are the revenues when 2, 14, 8 items, respectively, are sold.

f. When $x=10$ (10 units sold), the revenue is \$25. The price would be $p = 5 - \frac{10}{4} = \$2.50$

10. $A(x) = 50x - x^2$ $\quad A(1) = 49 \quad A(10) = 400 \quad A(20) = 600$

$A(25) = 625 \quad A(35) = 525$

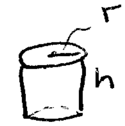
It appears as though the width of 25 feet will yield the largest area.

13. If x is length of a side of an equilateral triangle we get

Area $= \frac{1}{2}$ base $\cdot$ height. Since base also x we get 2 parts each of length $(\frac{1}{2}x)$. To find h use Pythagorean Theorem

$h^2 + (\frac{1}{2}x)^2 = x^2 \Rightarrow h = \sqrt{x^2 - \frac{1}{4}x^2} = \sqrt{\frac{3}{4}x^2} = \frac{\sqrt{3}}{2}x$

$A(x) = \frac{1}{2}\frac{\sqrt{3}}{2}x \cdot x = \frac{\sqrt{3}}{4}x^2 \quad D: [0, \infty)$



14. Let h be height, r be radius and V is volume

$V = $ (area of top)(height) $= \pi r^2 h$. Given $h = 2r$ so

$V(r) = 2\pi r^3 \quad D: [0, \infty)$

15. $V = 2\pi r^3$ solve for $r \Rightarrow r^3 = \frac{V}{2\pi} \Rightarrow r = \sqrt[3]{\frac{V}{2\pi}}$ so

$r(V) = \sqrt[3]{\frac{V}{2\pi}} \quad D: [0, \infty)$

16. h is height, r radius, V volume, S surface area $S = 14$ (given)

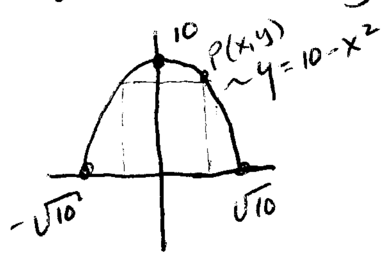
$S = 2\pi r^2 + 2\pi rh \Rightarrow 2\pi r^2 + 2\pi rh = 14$ and solve for $h \Rightarrow 2\pi rh = 14 - 2\pi r^2 \Rightarrow h = \frac{14 - 2\pi r^2}{2\pi r} = \frac{7 - \pi r^2}{\pi r}$

$V(r) = \pi r^2 h = \pi r^2\left(\frac{7 - \pi r^2}{\pi r}\right) = r(7 - \pi r^2) = 7r - \pi r^3$

17. $S = 4\pi r^2$ solve for r to get $r = \sqrt{\frac{S}{4\pi}}$. Since $V = \frac{4}{3}\pi r^3$

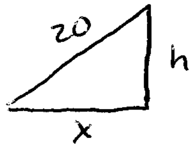
$V(S) = \frac{4}{3}\pi\left(\sqrt{\frac{S}{2\pi}}\right)^3 = \frac{4\pi S\sqrt{S}}{3(4\pi)\sqrt{4\pi}} = \frac{S\sqrt{S}}{6\sqrt{\pi}} \quad D: [0, \infty)$

18. A is area of rectangle $P(x, y)$ is point on curve $y = 10 - x^2$ and $A = $ length $\cdot$ width $= 2x(10 - x^2) = 20x - 2x^3$



$A(x) = 20x - 2x^3$

$D: [-\sqrt{10}, \sqrt{10}]$

19. 

A is area

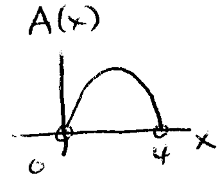
$x^2+h^2=20^2$

$h=\sqrt{400-x^2}$

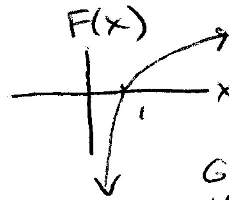
$A(x)=\frac{1}{2}x\sqrt{400-x^2}$

D: $(0,20)$

22. length is l. 500 feet of fencing is available get
$2x+l=500 \Rightarrow l=500-2x \quad A(x)=x(500-2x)=500x-2x^2$
D: $[0,250]$



26. $A(x)=8x-\frac{1}{2}x^3$ a) not a quadratic
b. one turning point. c) max at $\sim(2.31,12.32)$
no min



30. a) $F(x)=\sqrt{x}-\frac{1}{x}$ (a) not quadratic
(b) not turning points (c) no max or min

b) $G(x)=\sqrt{x}+\frac{1}{x}$ (a) not quadratic
(b) one turning point (c) no max min $\sim(1.587, 1.89)$

32. x and $4-x$ are length of the 2 pieces. So sides of squares are $\frac{x}{4}$ and $\frac{4-x}{4} \Rightarrow A(x)=\left(\frac{x}{4}\right)^2+\left(\frac{4-x}{4}\right)^2=\frac{1}{16}x^2+\frac{1}{16}(16-8x+x^2)$
$=\frac{1}{16}x^2+1-\frac{1}{2}x+\frac{1}{16}x^2=\frac{1}{8}x^2-\frac{1}{2}x+1$ D: $(0,4)$

33. for square side is $\frac{x}{4}$ Rectangle with perimeter $3-x$:
$2w+2l=3-x \Rightarrow 2(\frac{1}{2}l)+2l=3-x \Rightarrow 3l=3-x$ So
$l=1-\frac{1}{3}x$ and $w=\frac{1}{2}-\frac{1}{6}x \quad A(x)=\left(\frac{x}{4}\right)^2+\left(1-\frac{1}{3}x\right)\left(\frac{1}{2}-\frac{1}{6}x\right)=$
$\frac{1}{16}x^2+\frac{1}{2}-\frac{1}{3}x+\frac{1}{18}x^2=\frac{17}{144}x^2-\frac{1}{3}x+\frac{1}{2}$

34. a) $V=\frac{1}{3}\pi r^2h$ Given $V=12\pi\,cm^3 \Rightarrow 12\pi=\frac{1}{3}\pi r^2h \Rightarrow$
$h=\frac{12}{\frac{1}{3}\pi r^2}=\frac{36}{r^2} \quad h(r)=\frac{36}{r^2}$ D: $(0,\infty)$

b) from a $h=\frac{36}{r^2} \Rightarrow r=\sqrt{\frac{36}{h}} \quad r(h)=\frac{6}{\sqrt{h}} \quad (0,\infty)$

35. a) $V = \frac{1}{3}\pi r^2 h$ and $h = \sqrt{3}r$ $V = \frac{1}{3}\pi r^2(\sqrt{3}r) =$
$\frac{\sqrt{3}}{3}\pi r^3$ $V(r) = \frac{\sqrt{3}}{3}\pi r^3$ D: $(0, \infty)$

b) $S = \pi r\sqrt{r^2+h^2}$ and $h = \sqrt{3}r$ $S(r) = \pi r\sqrt{r^2+(\sqrt{3}r)^2}$
$= \pi r\sqrt{r^2+3r^2} = \pi r\sqrt{4r^2} = \pi r(2r) = 2\pi r^2$ D: $(0, \infty)$

39. x is length of wire used for circle and $14-x$ is length of wire used for square $\Rightarrow 2\pi r = x \Rightarrow r = \frac{x}{2\pi}$
$A = \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}$ for circle and
$4s = 14-x \Rightarrow s = \frac{14-x}{x}$ so $A = \left(\frac{14-x}{4}\right)^2 = \frac{(14-x)^2}{16}$ for square
$A(x) = \frac{x^2}{4\pi} + \frac{(14-x)^2}{16} = \frac{4x^2+\pi(14-x)^2}{16\pi}$ D: $(0, 14)$

41. Perimeter of each semicircle is $\frac{1}{2}(2\pi r) = \pi r$ so total perimeter P is $P = \pi r + \pi r + l + l = 2\pi r + 2l$ where l is length of rectangle since $P = \frac{1}{4} \Rightarrow 2\pi r + 2l = \frac{1}{4} \Rightarrow$
$2l = \frac{1}{4} - 2\pi r \Rightarrow 2l = \frac{1-8\pi r}{4} \Rightarrow l = \frac{1-8\pi r}{8}$. To find area: area of each semi-circle is $\frac{1}{2}\pi r^2$ and area of rectangle is $l \cdot w$ where $w = 2r$ so $A = \frac{1}{2}\pi r^2 + \frac{1}{2}\pi r^2 + lw =$
$\pi r^2 + \frac{1-8\pi r}{8}(2r) = \pi r^2 + \frac{r-8\pi r^2}{4} = \frac{4\pi r^2 + r - 8\pi r^2}{4}$
so $A(r) = \frac{r-4\pi r^2}{4}$ D: $\left(0, \frac{1}{4\pi}\right)$

42. If r is radius By Pythagorean Theorem $x^2+x^2 = (2r)^2$
$\Rightarrow 2x^2 = 4r^2 \Rightarrow r^2 = \frac{x^2}{2}$ $A = \pi r^2 \Rightarrow A(x) = \frac{\pi x^2}{2}$ D: $(0, \infty)$

44. a) Let $BC = x$ Cost $= 8000(AC) + 2000(CD)$. Use Pythagorean Thm where $d = AC$ so $10^2 + x^2 = d^2 \Rightarrow$
$d = \sqrt{100+x^2}$. Since $CD = 50-x$ Cost $= 8000\sqrt{100+x^2} + 2000(50-x)$
$C(x) = 8000\sqrt{100+x^2} + 10{,}000 - 2000x$ D: $[0, 50]$

b)

X(miles)	0	10	20	30	40	50
Cost	180,000	193,100	238,900	293,000	349,800	407,900

c) Cost appears to e increasing $0 \le x \le 50$. Lowest cost when $x=0$

d)

X(miles)	0	4	8	12	16	20
Cost	180,000	178,200	186,400	201,000	218,900	238,900

e) cost not increasing on $0 \le x \le 20$. Lowest cost at $x=4$

45. a) $V = lwh$ $\quad l = 8-2x \quad w = 6-2x \quad h = x$

$V = (8-2x)(6-2x)(x) \Rightarrow V(x) = 4x^3 - 28x^2 + 48x$ D: $(0,3)$

b)

X(ins)	0	0.5	1.0	1.5	2.0	2.5	3.0
Vol (in³)	0	17.5	24	22.5	16	7.5	0

c) $x=1$ yields largest volume

d)

X(in)	0.8	0.9	1.0	1.1	1.2	1.3	1.4
V(in³)	22.5	23.4	24	24.2	24.2	23.9	23.3

e) $x = 1.1$ yields largest volume

46. $V = lwh$ $\quad l = 12-2x \quad w = 12-2x \quad h = x$

$V = (12-2x)^2(x)$ So $V(x) = 4x^3 - 48x^2 + 144x$

D: $(0,6)$

47. a) $A = \frac{1}{2}(\pi r^2) + lw$ $\quad w = 2r$ and Perimeter $= 32$ ft

$P = \frac{1}{2}(2\pi r) + 2l + 2w$ Solve for l $\quad l = \frac{32 - \pi r - 2r}{2}$

$A = \frac{1}{2}(\pi r^2) + \left(\frac{32 - \pi r - 2r}{2}\right)(2r) = 32r - 2r^2 - \frac{\pi r^2}{2}$

D: $\left(0, \frac{32}{\pi + 2}\right)$

50. a) Let $y=0$ $\quad 16-x^2=0 \quad x^2=16 \quad x=4$ (since $x \geq 0$)

b) Since $0<x<4$ is in the 1st quadrant, the domain of function is $(0,4)$

52. If x is the east-west dimension and y is the north-south dimension then Cost is given by

$C = 12(2x)+8(2y) = 24x+16y$ Since cost is \$4800

$24x+16y = 4800$ so $y = \frac{4800-24x}{16} = \frac{600-3x}{2}$

$A = xy = x\left(\frac{600-3x}{2}\right) = -\frac{3}{2}x^2+300x$ so

$A(x) = -\frac{3}{2}x^2+300x$ D: $(0,200)$. Since this is a parabola pointing down, the max occurs at the vertex (find by completing square)

$A(x) = -\frac{3}{2}x^2+300x = -\frac{3}{2}(x^2-200x) =$

$-\frac{3}{2}(x^2-200x+100^2-100^2) = -\frac{3}{2}(x^2-200x+100^2)+15{,}000$

$= -\frac{3}{2}(x-100)^2+15{,}000$. So the length of 100 yards will give a maximum area of 15,000 yd^2