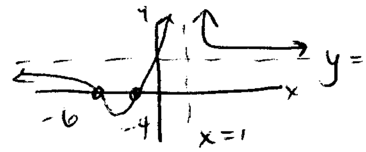
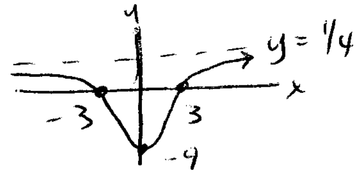
Section 4.7 Rational Functions - Solutions.

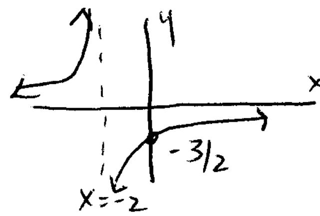
2. $y = \dfrac{(x+6)(x+4)}{(x-1)^2}$ HA: $y=1$
x-int $(-6,0)$ $(-4,0)$
y-int. $(0,24)$
D: $x \neq 1 \Rightarrow$ VA $x=1$



6. $y = \dfrac{x^2-9}{4x^2+1}$ HA: $y = \frac{1}{4}$
x-int: $(-3,0)(3,0)$
D: all reals $\Rightarrow$ no VA y-int $(0,-9)$

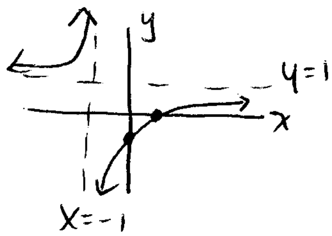


12. $y = \dfrac{-3}{x+2}$ VA: $x=-2$
HA: $y=0$
x-int: none
y-int: $(0,-3/2$
D: $\mathbb{R}$ exc. $x=-2$

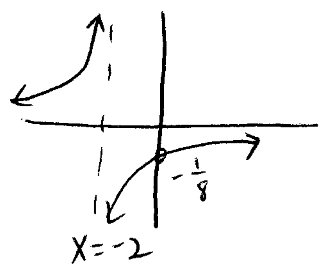


this is a horizontal shift 2 units to left, reflect across horizontal axis, vertical stretch of 3 of basic graph $y=\frac{1}{x}$

14. $y = \dfrac{x-1}{x+1}$ VA: $x=-1$
HA $y=1$
x-int $(1,0)$
y-int $(0,-1)$
D: $\mathbb{R}$ exc $x=-1$

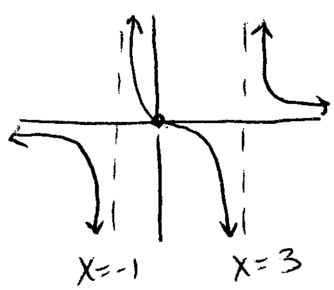


24. $y = \dfrac{-1}{(x+2)^3}$ VA: $x=-2$
HA $y=0$
x-int: none
y-int: $(0,-\frac{1}{8})$
D: $\mathbb{R}$ exc $x=-2$

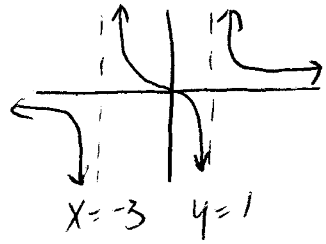


this is also a translation of $y=\frac{1}{x}$

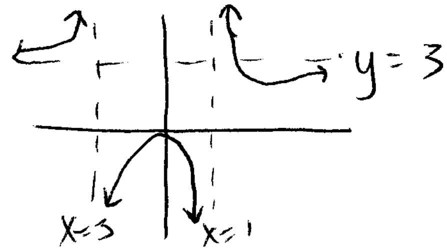
26. $y = \dfrac{x}{(x+1)(x-3)}$ VA: $x=-1$
$x=3$
D: $\mathbb{R}$ exc $x=-1,3$
HA: $y=0$
x-int: $(0,0)$
y-int: $(0,0)$



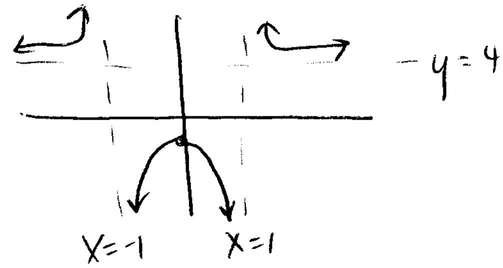
29. a) $y = \dfrac{3x}{(x-1)(x+3)}$ VA: $x=1$
$x=-3$
HA $y=0$
x-int $(0,0)$
y-int
D: $\mathbb{R}$ exc $x=1,-3$



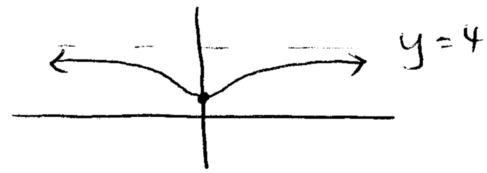
b) $y = \dfrac{3x^2}{(x-1)(x+3)}$ everything is same as a except HA: $y=3$



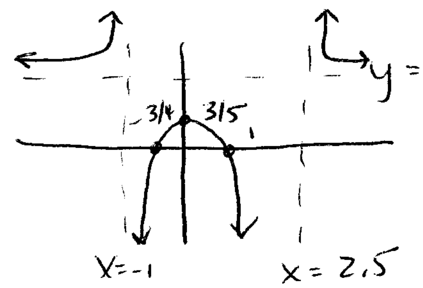
30. a) $y = \dfrac{4x^2+1}{x^2-1}$ HA $y=4$
x-int: none
D: $x \neq \pm 1$ VA $x = \pm 1$ y-int $(0,-1)$



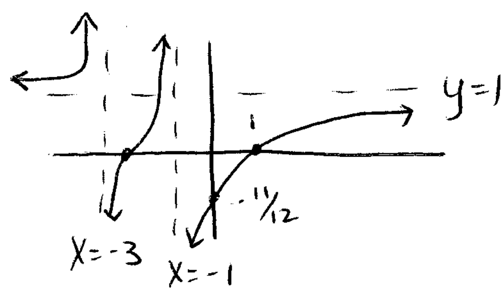
b). $y = \dfrac{4x^2+1}{x^2+1}$ HA : $y=4$
x-int: none
D: $\mathbb{R}$ VA: none y-int $(0,1)$



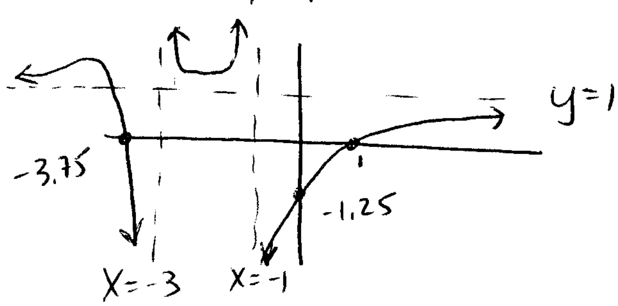
32. $y = \dfrac{4x^2-x-3}{2x^2-3x-5} = \dfrac{(4x+3)(x-1)}{(2x-5)(x+1)}$
D: $x \neq 5/2, -1$ VA: $x = \frac{5}{2}$ $x=-1$
HA : $y=2$ x-int $(-\frac{3}{4},0)(1,0)$ y-int $(0, 3/5)$



34. a) $f(x) = \dfrac{(x-1)(x+2.75)}{(x+1)(x+3)}$
D: $x \neq -1,-3$ VA $x=-1$, $x=-3$ y-int. $(0, -\frac{11}{12})$
HA: $y=1$ x-int $(1,0)$ $(-2.75,0)$

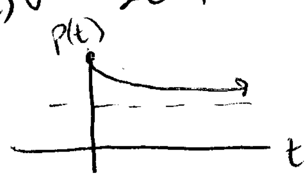


b) $g(x) = \dfrac{(x-1)(x+3.75)}{(x+1)(x+3)}$
Domain, HA and VA are same
x-int $(1,0)$ $(-3.75,0)$ y-int $(0,-1.25)$



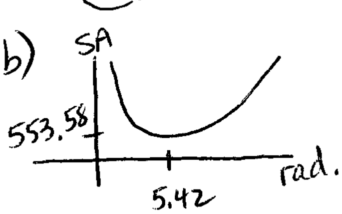
Where the zeros fell changed the graph.

36. $y = \dfrac{6t+12}{2t+1}$ $t \geq 0$ $P(0) = 12$ - there are initially 12 thousand bacteria in the colony
$P(t)$
$t \to \infty$ $P(t) \to 3$ (HA) - bacteria will eventually by 3000 bacteria. Obviously this is smaller.



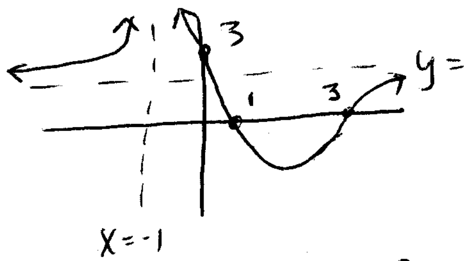
function is decreasing (bacteria is decreasing)

38.  $V = \pi r^2 h$ $1000 = \pi r^2 h \Rightarrow h = \dfrac{1000}{\pi r^2}$ a) $SA = 2\pi r^2 + 2\pi r h$
$= 2\pi r^2 + 2\pi r\left(\dfrac{1000}{\pi r^2}\right)$
$= 2\pi r^2 + \dfrac{2000}{r}$
$= \dfrac{2\pi r^3 + 2000}{r}$ $r > 0$

b) 

minimum SA is 553.58 cm^2 when radius is 5.42 cm and height is 10.84 cm

40. $y = \dfrac{(x-1)(x-3)}{(x+1)^2}$



D: $x \neq -1$ VA $x=-1$ HA $y=1$

x-int $(1,0)$ $(3,0)$ y-int $(0,3)$

$1 = \dfrac{(x-1)(x-3)}{(x+1)^2} \Rightarrow (x+1)^2 = (x-1)(x-3) \Rightarrow x^2+2x+1 = x^2-4x+3$

$\Rightarrow 6x = 2 \Rightarrow x = 1/3$ so the function crosses HA when $x = 1/3$

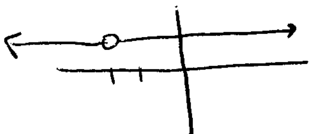
43. $y = \dfrac{(x-3)(x+2)}{(x+1)(x-2)}$

x close to -2 $y = \dfrac{(-2-3)(x+2)}{(-2+1)(-2-2)} = -\frac{5}{4}(x+2)$

x close to -1 $y = \dfrac{(-1-3)(-1+2)}{(x+1)(-1-2)} = \frac{4}{3}\left(\frac{1}{x+1}\right)$

x close to 2 $y = \dfrac{(2-3)(2+2)}{(2+1)(x-2)} = -\frac{4}{3}\left(\frac{1}{x-2}\right)$

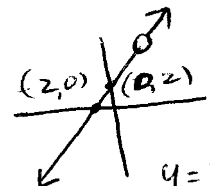
44. a) $y = \dfrac{x+2}{x+2} = 1$



with hole at $x=-2$ on y; $y=1$ Domain: all reals.

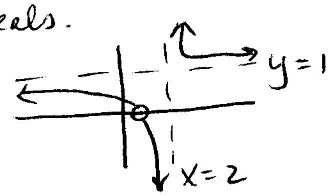
b. $y = \dfrac{x^2-4}{x-2} = x+2$

with hole at $x=2$



$y=x+2$ has domain all reals

c) $y = \dfrac{x-1}{(x-1)(x-2)} = \dfrac{1}{x-2}$



hole at $x=1$

$y = \frac{1}{x-2}$ has D: $x \neq 2$

49. a)

$$\begin{array}{r} x+4 \\ x-3 \overline{\smash{)}\, x^2+x-6} \\ \underline{x^2-3x} \\ 4x-6 \\ \underline{4x-12} \\ 6 \end{array}$$

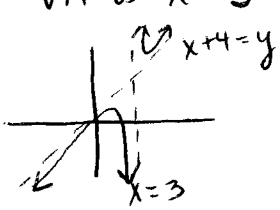
$F(x) = \dfrac{x^2+x-6}{x-3} = (x+4) + \dfrac{6}{x-3}$

b)

x	$x+4$	$\frac{x^2+x-6}{x-3}$	x	$x+4$	$\frac{x^2+x-6}{x-3}$
10	14	14.857	-10	-6	-6.462
100	104	104.062	-100	-96	-96.058
1000	1004	1004.006	-1000	-996	-996.006

c). VA is $x=3$ x-int: $x^2+x-6 = (x+3)(x-2)$ so $(-3,0)$ $(2,0)$ y-int $(0,2)$

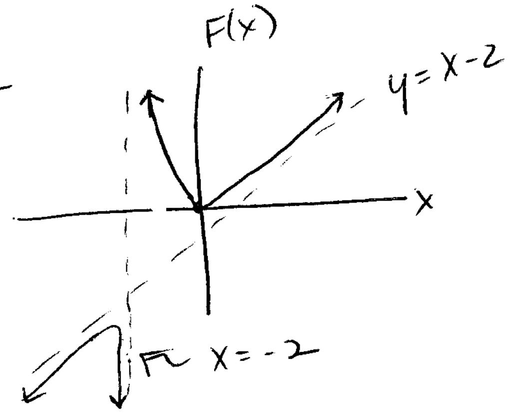
d)



50. $F(x) = \dfrac{x^2}{x+2}$

$$\begin{array}{r} x-2 \\ x+2\overline{)\,x^2} \\ \underline{x+2} \\ -2x \\ \underline{-2x-4} \\ 4 \end{array}$$

$= (x-2) + \dfrac{4}{x+2}$



So slant asymptote is $y = x-2$

D: $x \neq -2$ VA $x = -2$

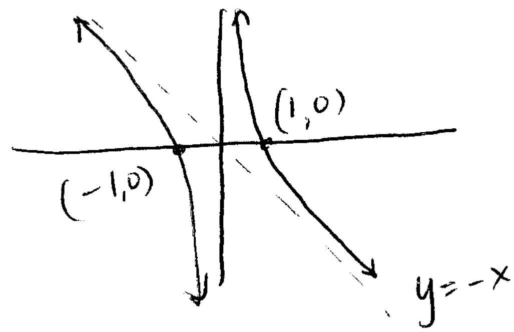
x-int $(0,0)$

y-int $(0,0)$

51. $y = \dfrac{1-x^2}{x}$

$$\begin{array}{r} -x \\ x\overline{)\,-x^2+1} \\ \underline{-x^2} \\ 1 \end{array}$$

$= -x + \dfrac{1}{x}$



So slant asymptote is $y = -x$

D: $x \neq 0$

VA: $x = 0$

x-int: $1-x^2 = (1-x)(1+x)$

So $(1,0)$ $(-1,0)$