Section 5.3 Logarithmic Functions - Solutions.

2. $f(g(3)) = 3$ $g(f(5)) = 5$

4. a) $f(x) = e^x$ $g(x) = \ln x$ is the inverse function

b) $e^{\ln x} = x$ $\ln e^x = x$

c) $2^{\log_2 99} = 99$ $\log_2 2^{-\pi} = -\pi$

6. $f(x) = 4x - 5$ a) $f^{-1}(x) = \frac{x+5}{4}$ [$y = 4x-5 \Rightarrow y+5 = 4x \Rightarrow x = \frac{y+5}{4}$]

b) $f(f^{-1}(x)) = x$ c) $f(x) f^{-1}(x) = (4x-5)\left(\frac{x+5}{4}\right) = \frac{1}{4}(4x^2+15x-25)$

$= x^2 + \frac{15}{4}x - \frac{25}{4}$

d) b is involved in the definition of an inverse function.

7. $f(x) = x^3 + 5x + 1$ $f(f^{-1}(6)) = 6$

10. a) $1/125 = 5^{-3} \Rightarrow \log_5 \frac{1}{125} = -3$

b) $e^0 = 1 \Rightarrow \ln 1 = 0$

c) $5^x = 6 \Rightarrow \log_5 6 = x$

d) $e^{3t} = 8 \Rightarrow \ln 8 = 3t$

11. a) $\log_2 32 = 5 \Rightarrow 2^5 = 32$ b) $\log_{10} 1 = 0 \Rightarrow 10^0 = 1$

c) $\log_e \sqrt{e} = \frac{1}{2} \Rightarrow e^{1/2} = \sqrt{e}$ d) $\ln\left(\frac{1}{e}\right) = -1 \Rightarrow e^{-1} = \frac{1}{e}$

12. a) $\ln u = s \Rightarrow e^s = u$ b) $\log_a b = c \Rightarrow a^c = b$

18. a) $\log_{25}\left(\frac{1}{625}\right) = -2$ because $25^{-2} = \frac{1}{625}$

b) $\log_{16} \frac{1}{64} = -3/2$ because $16^{-3/2} = (\sqrt{16})^{-3} = 4^{-3} = \frac{1}{64}$

c) $\log_{10} 10 = 1$ because $10^1 = 10$

d) $\log_2 8\sqrt{2} = 3.5$ because $2^{7/2} = (\sqrt{2})^7 = \sqrt{2^7} = \sqrt{2^6 2^1} = 8\sqrt{2}$

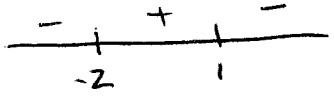
20. a) $\log_5 x = e \Rightarrow 5^e = x \approx 79.43$ b) $\ln x = e \Rightarrow e^{-e} = x \approx .07$

21. a) $y = \log_4 5x$ D: $x > 0$ b) $y = \log_{10}(3-4x)$ D: $x < 3/4$

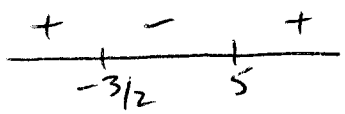
c) $y = \ln(x^2)$ D: $x \neq 0$ d. $y = (\ln x)^2$ D: $x > 0$ e. $y = \ln(x^2 - 25)$ D: $x < -5$ OR $x > 5$

22. a) $y = \ln(2 - x - x^2)$ D: $-2 < x < 1$

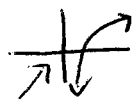
$-(x^2 + x - 2) > 0 \Rightarrow -(x+2)(x-1) > 0$

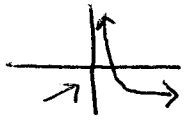


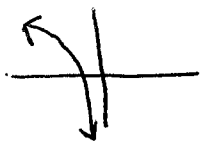
b) $y = \log \frac{2x+3}{x-5}$ D: $x < -3/2$ OR $x > 5$

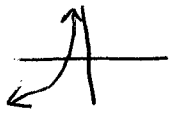


$\frac{2x+3}{x-5} > 0$

26. a $y = \ln x$  D: $x>0$ R: y is any real x-int $(1,0)$ VA of $x=0$

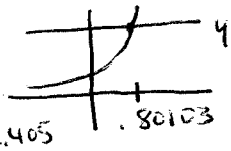
b. $y = -\ln x$  D: $x>0$ R: $\mathbb{R}$ x-int $(1,0)$ VA of $x=0$

c. $y = \ln(-x)$  D: $x<0$ R: $\mathbb{R}$ x-int $(-1,0)$ VA of $x=0$

d) $y = -\ln(-x)$  D: $x<0$ R: $\mathbb{R}$ x-int $(-1,0)$ VA of $x=0$

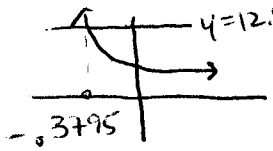
31. a) $\ln e^4 = 4$ b) $\ln(\frac{1}{e}) = \ln e^{-1} = -1$ c) $\ln\sqrt{e} = \ln e^{1/2} = \frac{1}{2}$

32. a) $\ln e = 1$ b) $\ln e^{-2} = -2$ c) $(\ln e)^{-2}\left(\frac{1}{\ln e}\right)^2 = 1$

36. $(10^x)^2 = 40 \Rightarrow 10^{2x} = 40$  $y=40$, $.80103$

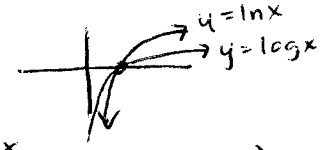
$10^{2x} = 40 \Rightarrow \log 10^{2x} = \log 40$

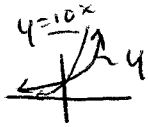
$\Rightarrow 2x = \log 40 \Rightarrow x = \frac{\log 40}{2} \approx .80103$

39. $e^{1-4t} = 12.405$  $y=12.405$, $-.3795$

$\ln e^{1-4t} = \ln 12.405 \Rightarrow 1-4t = \ln 12.405$

$t = \frac{(\ln 12.405) - 1}{-4} \approx -.3795$

41. a) $\ln x < \log_{10} x$ $0<x<1$  $y=\ln x$, $y=\log x$ b) $\ln x > \log x$ $x>1$

c) $e^x < 10^x$  $y=10^x$, $y=e^x$ $x>0$ d) $e^x > 10^x$ $x<0$

43. $m = \log_{10}\left(\frac{A}{A_0}\right) \Rightarrow A = A_0 10^M$ so $A = A_0 10^{6.4}$ $A = A_0 10^{6.6}$

$\frac{\text{A-value San Salvador}}{\text{A-value India}} = \frac{A_0 10^{6.6}}{A_0 10^{6.4}} = 10^{.2} \approx 1.58$ is magnitude difference

50. 3.5×10^{-9} pH = 9 base

51 pH is 5.9 $10^{-5.9}$

54. $y = \ln(-x)$ D, E, H

56. $y = \ln|x|$ E, H

57. $y = \ln x + e$ D, E, H

58. $y = x + \ln e$ A, D, F, G, H

60. $y = (\ln x)^2$ C, E

62. $g(t) = \ln(t-1)$ inverse $y = \ln(t-1) \Rightarrow e^y = t-1 \Rightarrow t = e^y + 1 \Rightarrow$ $f^{-1}(t) = e^t + 1$

 vert. int $(0,2)$ x-int none HA $y=1$

69. $f(x) = \ln(\ln x)$ a) $\ln x > 0$ so Domain: $x>1$

b. $y = \ln(\ln x) \Rightarrow e^y = \ln x \Rightarrow e^{e^y} = x \Rightarrow f^{-1}(x) = e^{e^x}$