# Section 5.4 - Properties of Logarithms - Solutions

2. $\log_{10}(40) + \log_{10}(5/2) = \log_{10}\left(40 \cdot \frac{5}{2}\right) = \log_{10} 100 = \log_{10} 10^2 = 2$

4. $\log_9 25 - \log_9 75 = \log_9\left(\frac{25}{75}\right) = \log_9\left(\frac{1}{3}\right) = \log_9\left(9^{-1/2}\right) = -\frac{1}{2}$

6. $\ln e^3 - \ln e = 3\ln e - \ln e = 3 - 1 = 2$

8. $e^{\ln 3} + e^{\ln 2} - e^{\ln e} = 3 + 2 - e = 5 - e$

10. $\log_b b^6 = b$

14. $p\log_b A - q\log_b B + r\log_b C = \log_b A^p - \log_b B^q + \log_b C^r = \log_b\left(\frac{A^p C^r}{B^q}\right)$

16. a) $\log_{10}(x^2-16) - 3\log_{10}(x+4) + 2\log_{10} x =$

$\log_{10}(x-4)(x+4) - \log_{10}(x+4)^3 + \log_{10} x^2 = \log_{10}\left(\frac{x^2(x-4)(x+4)}{(x+4)^3}\right) = \log_{10}\left(\frac{x^2(x-4)}{(x+4)^2}\right)$

b) $\log_{10}(x^2-16) - 3\left[\log_{10}(x+4) + 2\log_{10} x\right]$

$= \log_{10}(x^2-16) - 3\log_{10}(x+4) - 6\log_{10} x = \log_{10}\left(\frac{(x-4)(x+4)}{(x+4)^3 x^6}\right) = \log_{10}\left(\frac{x-4}{x^6(x+4)^2}\right)$

18. $\ln(x^3-1) - \ln(x^2+x+1) = \ln\left(\frac{x^3-1}{x^2+x+1}\right) = \ln(x-1)$

23. a) $\log_{10}\sqrt{9-x^2} = \log_{10}(9-x^2)^{1/2} = \frac{1}{2}\log_{10}(3-x) + \frac{1}{2}\log_{10}(3+x)$

b) $\ln\frac{\sqrt{4-x^2}}{(x-1)(x+1)^{3/2}} = \frac{1}{2}\ln(4-x^2) - \ln(x-1) - \frac{3}{2}\ln(x+1)$

$= \frac{1}{2}\ln(2-x) + \frac{1}{2}\ln(2+x) - \ln(x-1) - \frac{3}{2}\ln(x+1)$

26. a) $\log_b \sqrt[3]{\frac{(x-1)^2(x-2)}{(x+2)^2(x+1)}} = \frac{1}{3}\left[2\log_b(x-1) + \log_b(x-2) - 2\log_b(x+2) - \log_b(x+1)\right]$

$= \frac{2}{3}\log_b(x-1) + \frac{1}{3}\log_b(x-2) - \frac{2}{3}\log_b(x+2) - \frac{1}{3}\log_b(x+1)$

b. $\ln\left(\frac{e-1}{e+1}\right)^{3/2} = \frac{3}{2}\left[\ln(e-1) - \ln(e+1)\right] = \frac{3}{2}\ln(e-1) - \frac{3}{2}\ln(e+1)$

30. $\log_b 2 = A \quad \log_b 3 = B \quad \log_b 5 = C$

a). $\log_b\sqrt{5} = \frac{1}{2}\log_b 5 = \frac{1}{2}C$ b) $\log_b\sqrt{15} = \frac{1}{2}\log_b(5\cdot 3) = \frac{1}{2}\left[\log_b 5 + \log_b 3\right] = \frac{1}{2}(A+C)$

c) $\log_b\sqrt[3]{0.4} = \frac{1}{3}\log_b\left(\frac{2}{5}\right) = \frac{1}{3}\left[\log_b 2 - \log_b 5\right] = \frac{1}{3}(A-C)$

d) $\log_b\sqrt[4]{60} = \frac{1}{4}\left[\log_b(2\times 3\times 2\times 5)\right] = \frac{1}{4}\left[\log_b 2 + \log_b 3 + \log_b 2 + \log_b 5\right] = \frac{1}{4}(2A+B+C)$

38. $\log_{10} A = a \quad \log_{10} B = b \quad \log_{10} C = c$

a) $\log_{10} A + 2\log_{10}\left(\frac{1}{A}\right) = \log_{10} A + 2\log_{10}(A^{-1}) = a - 2a = -a$

b) $\log_{10}(A/10) = \log_{10} A - \log_{10} 10 = a - 1$

c) $\log_{10}\frac{100A^2}{B\sqrt[3]{C}} = \log_{10} 10^2 + 2\log_{10} A - 4\log_{10} B - \frac{1}{3}\log_{10} C = 2 + 2a - 4b - \frac{1}{3}C$

d) $\log_{10}\frac{(4B)^5}{C} = 5\log_{10} 4B - \log_{10} C = 5\log_{10} 4 + 5\log_{10} B - \log_{10} C = 5\log_{10} 4 + 5b - C$

43. $5 = 2e^{2x-1} \Rightarrow \ln\frac{5}{2} = \ln e^{2x-1} \Rightarrow 2x-1 = \ln\frac{5}{2} \Rightarrow 2x = \ln\frac{5}{2}+1$

$\Rightarrow x = \frac{\ln 2.5 + 1}{2}$ or $\frac{\ln 5 - \ln 2 + 1}{2}$

46. $\log 5^{3x-1} = \log 27 \Rightarrow (3x-1)\log 5 = \log 27 \Rightarrow 3x\log 5 - \log 5 = \log 27 \Rightarrow$

$3x\log 5 = \log 27 + \log 5 \Rightarrow x = \frac{\log 27 + \log 5}{3\log 5}$ or $\frac{3\log 3 + \log 5}{3\log 5}$

44. $3e^{1+t} = 2 \Rightarrow \ln e^{1+t} = \ln 2/3 \Rightarrow (1+t) = \ln(2/3) \Rightarrow t = \ln\left(\frac{2}{3}\right) - 1$

50. $2^{9-x^2} = 430.5389 \Rightarrow 9-x^2 = \frac{\log(430.5389)}{\log 2} \Rightarrow x^2 = 9 - \frac{\log(430.5389)}{\log 2}$

$\Rightarrow x = \pm\sqrt{9 - \frac{\log(430.5389)}{\log 2}} \approx \pm .500000216$5 (round $\pm .500$)

52. $\log_5 10 = \frac{\log 10}{\log 5} = \frac{1}{\log 5}$ 59. $\log_{10} e = \frac{\ln e}{\ln 10} = \frac{1}{\ln 10}$

63. a) T - prop. of addition b) T $\log_e \sqrt{e} = \frac{1}{2}\log_e e = \frac{1}{2}\ln e = \frac{1}{2}$

c) T - from b d) F $\ln x^3 = 3\ln x$ e) T - from d

f) F - $\ln 2x^3 = \ln 2 + 3\ln x$ g) T - $\log_a c = b \Leftrightarrow a^b = c$

h) F - $\log_5 24 = x \Rightarrow 5^x = 24$ $1 < x < 2$ i) T - see h

j) F - $\log_5 24$ is closer to 2 $5^2 = 25$ k) F D: $x > 0$ l) T R: $\mathbb{R}$

m) T - $y = e^x$ is its inverse

64. a) $b=10$ $P=\pi$ $Q=\sqrt{2}$ #2 $\log_b PQ = \log_b P + \log_b Q \Rightarrow \log_{10}(\sqrt{2}\cdot\pi) \approx$

$\approx .64766$ $\log\pi + \log\sqrt{2} \approx .49714987 + .1505149 \approx .64766$

b) $P=3$ $Q=4$ $\ln(3+4) = \ln 7 \approx 1.9459$ $\ln 3 + \ln 4 \approx 1.0986 +$

$1.38629 = 2.4849$

c) $b=10$ $P=2$ $Q=3$ #3 $\log_b\left(\frac{P}{Q}\right) = \log_b P - \log_b Q$

$\log_{10}\left(\frac{2}{3}\right) \approx -.17609$ $\log_{10} 2 - \log_{10} 3 \approx .30103 - .477121 = .17609$

d) $P=10$ $Q=20$ $\ln(200) \approx 5.2983$ $(\ln 10)(\ln 20) \approx (2.3025)(2.9957)$

$= 6.8979$

e) $P=17$ $Q=76$ $\ln\left(\frac{17}{76}\right) \approx -1.4975$

$\ln 17 - \ln 76 \approx -1.4975$

f) $\frac{\log 17}{\log 76} \approx .6542$ $\log 17 - \log 76 \approx -1.4975$

69. $(-2, 324)\left(\frac{1}{2}, \frac{4}{3}\right) \Rightarrow 324 = ae^{-2b}$ $\frac{4}{3} = ae^{\frac{1}{2}b} \Rightarrow$

$\frac{324}{e^{-2b}} = \frac{4/3}{e^{\frac{1}{2}b}} \Rightarrow \frac{e^{\frac{1}{2}b}}{e^{-2b}} = \frac{4/3}{324} \Rightarrow e^{2.5b} = \frac{1}{243} = (243)^{-1}$

$\frac{5}{2}b = -\ln(243) \Rightarrow b = -\frac{2}{5}\ln(243) = -\ln 9$ $\frac{4}{3} = ae^{-\frac{\ln 9}{2}} = ae^{\ln\frac{1}{3}}$

$\frac{1}{3}a = 4/3$

$a = 4$

$y = 4e^{(-\ln 9)x}$

70. $(1,2)$ and $(4,8) \Rightarrow 2 = ae^{b(1)}$ and $8 = ae^{8b}$

$\Rightarrow a = 2e^{-b} \Rightarrow 8 = 2(e^{-b})(e^{8b}) \Rightarrow \ln 4 = \ln e^{7b} \Rightarrow$

$7b = \ln 4 \Rightarrow b = \frac{\ln 4}{7}$ $a = 2e^{-\frac{\ln 4}{7}} = 2e^{-\frac{1}{7}\ln 4} \Rightarrow 2e^{\ln 4^{-1/7}}$

$= 2(4^{-1/7})$

75. $b^{3\log_b x} = b^{\log_b x^3} = x^3$