Section 5.5 - Equations with Logs + Exponents - Solutions

1. $5^x = 3^{2x-1} \Rightarrow x\ln 5 = (2x-1)\ln 3 \Rightarrow x\ln 5 = 2x\ln 3 - \ln 3 \Rightarrow \ln 3 = 2x\ln 3 - x\ln 5$
$\Rightarrow x(2\ln 3 - \ln 5) = \ln 3 \Rightarrow \quad x = \dfrac{\ln 3}{2\ln 3 - \ln 5} \approx 1.869$

2. $7^{-4x} = 2^{1+3x} \Rightarrow -4x\log 7 = (1+3x)\log 2 \Rightarrow -4x\log 7 = \log 2 + 3x\log 2$
$\Rightarrow -\log 2 = 3x\log 2 + 4x\log 7 \Rightarrow x(3\log 2 + 4\log 7) = -\log 2 \Rightarrow$
$x = \dfrac{-\log 2}{3\log 2 + 4\log 7} \approx -.070$

3. $\ln(\ln x) = 1.5 \Rightarrow \ln x = e^{1.5} \Rightarrow x = e^{e^{1.5}} \approx 88.384$

5. $\log_{10}(x^2+36) = 2 \Rightarrow x^2+36 = 100 \Rightarrow x^2 = 64 \Rightarrow x = \pm 8$

8. $\log_9(x^2+x) = 0.5 \Rightarrow x^2+x = 9^{1/2} \Rightarrow x^2+x-3=0 \quad x = \dfrac{-1 \pm \sqrt{1^2-4(1)(-3)}}{2(1)}$
$= \dfrac{-1\pm\sqrt{13}}{2} \approx 1.303, -2.303$ which are both in domain

9. $10^{2x} + 3(10^x) - 10 = 0$ if $t = 10^x \Rightarrow t^2 + 3t - 10 = 0 \Rightarrow (t+5)(t-2) = 0$
$t = -5, 2 \Rightarrow 10^x = -5$ (no solution) $10^x = 2 \Rightarrow x = \log 2 \approx .301$

10. $3(2^{2x}) - 11(2^x) - 4 = 0$ if $u = 2^x \Rightarrow 3u^2 - 11u - 4 = 0 \Rightarrow (3u+1)(u-4) = 0$
$\Rightarrow u = -1/3, 4 \Rightarrow 2^x = -1/3$ (no Solution) $2^x = 4 \Rightarrow x = 2$

12. a) $(\log_{10}x)^2 = 2\log_{10}x$ if $t = \log x \Rightarrow t^2 - 2t = 0 \Rightarrow t(t-2) = 0$
$\Rightarrow t = 0, t = 2 \Rightarrow \log x = 0 \Rightarrow x = 1 \quad \log x = 2 \Rightarrow x = 100$

b) $\log_{10}(x^2) = 2\log_{10}x \Rightarrow 2\log_{10}x = 2\log_{10}x$ on Domain: $x > 0$
So this is true when $x > 0$

15. $7^{\log_7 2x} = 2x \Rightarrow 2x = 2x$ true when $x > 0$

17. $7^{\log_7 2x} = 7 \Rightarrow 2x = 7 \Rightarrow x = 7/2$

18. $\ln(\ln(\ln x)) = 1 \Rightarrow \ln(\ln x) = e \Rightarrow \ln x = e^e \Rightarrow x = e^{e^e}$
≈ 3814279.105

26a) $4e^{6x} - 12e^{3x} + 9 = 0$ if $u = e^{3x} \Rightarrow 4u^2 - 12u + 9 = 0 \Rightarrow$
$(2u-3)^2 = 0 \Rightarrow u = 3/2 \Rightarrow e^{3x} = \frac{3}{2} \Rightarrow x = \frac{1}{3}\ln(1.5) \approx .135$

b) $4e^{6x} + 12e^{3x} + 9 = 0 \Rightarrow 4u^2 + 12u + 9 = 0 \Rightarrow (2u+3)^2 = 0$
$\Rightarrow u = -3/2 \Rightarrow e^{3x} = -3/2$ Which has no solution

26c) $4e^{6x}-16e^{3x}-9=0 \Rightarrow 4u^2-16u-9=0 \Rightarrow (2u-9)(2u+1)=0$

$\Rightarrow u=\frac{9}{2}\ \ u=-1/2 \Rightarrow e^{3x}=-\frac{1}{2}$ (no sol) $e^{3x}=\frac{9}{2} \Rightarrow x=\frac{\ln(9/2)}{2} \approx .501$

d) $e^{6x}-12e^{3x}-9=0 \Rightarrow u^2-12u-9=0$ using quad form $u=6\pm3\sqrt{5}$

$\Rightarrow e^{3x}=6-3\sqrt{5}$ (no sol) $e^{3x}=6+3\sqrt{5} \Rightarrow x=\frac{1}{3}\ln(6+3\sqrt{5}) \approx .847$

34. $\log_2(x+4)=2-\log_2(x+1) \Rightarrow \log_2(x+4)+\log_2(x+1)=2$

$\Rightarrow \log_2(x+4)(x+1)=2 \Rightarrow x^2+5x+4=4 \Rightarrow x^2+5x=0 \quad x=0, \cancel{-5}$

37. $\log_{10}(x+3)-\log_{10}(x-2)=2 \Rightarrow \log\frac{x+3}{x-2}=2 \Rightarrow \frac{x+3}{x-2}=100$

$\Rightarrow x+3=100(x-2) \Rightarrow x+3=100x-200 \Rightarrow 99x=203 \Rightarrow x=\frac{203}{99}$

38. $\ln(x+1)=2+\ln(x-1) \Rightarrow \ln(x+1)-\ln(x-1)=2 \Rightarrow$

$\ln\left(\frac{x+1}{x-1}\right)=2 \Rightarrow \frac{x+1}{x-1}=e^2 \Rightarrow (x+1)=e^2x-e^2 \Rightarrow x-e^2x=-1-e^2$

$\Rightarrow x(1-e^2)=-(1+e^2) \Rightarrow x=-\frac{1+e^2}{1-e^2}=\frac{1+e^2}{e^2-1} \approx 1.313$

42. $\log_2(x+a)-\log_2(x-a)=1 \Rightarrow \log_2\frac{x+a}{x-a}=1 \Rightarrow \frac{x+a}{x-a}=2$

$\Rightarrow x+a=2(x-a) \Rightarrow x+a=2x-2a \Rightarrow x-2x=-2a-2 \Rightarrow$

$-x=-3a \Rightarrow x=3a$

52. $6(5-1.6^x)\geq 13 \Rightarrow 5-1.6^x \geq \frac{13}{6} \Rightarrow -1.6^x \geq \frac{13}{6}-5 \Rightarrow$

$1.6^x \leq 5-\frac{13}{6} \Rightarrow x\ln 1.6 \leq \ln\left(\frac{17}{6}\right) \Rightarrow x \leq \frac{\ln 17/6}{\ln 1.6} \approx 2.216$

62. $\ln\left(\frac{3x-2}{4x+1}\right)>\ln 4 \Rightarrow \frac{3x-2}{4x+1}>4 \Rightarrow \frac{3x-2}{4x+1}-4>0 \Rightarrow$

$\frac{3x-2-4(4x+1)}{4x+1}>0 \Rightarrow \frac{-13x-6}{4x+1}>1$

$-\quad +\quad -$

$-\frac{6}{13}\quad -1/4$

$-\frac{6}{13}<x<-\frac{1}{4}$

68. a) $y=\ln x+\ln(x+2)$ D: $x>0$

b) $\ln x+\ln(x+2)\leq \ln 35 \Rightarrow \ln x(x+2)\leq \ln 35 \Rightarrow$

$x^2+2x\leq 35 \Rightarrow x^2+2x-35\leq 0 \Rightarrow (x+7)(x-5)\leq 0$

$+\quad -\quad +$

$-7\quad 5$

$-7\leq x\leq 5$ and domain: $x>0$

So $\boxed{0<x\leq 5}$

70. $\log_{10}(x^2-6x-6)>0 \Rightarrow x^2-6x-6>1 \Rightarrow x^2-6x-7>0$

$\Rightarrow (x-7)(x+1)>0$ $\quad \underset{-1}{+ \mid} \; - \; \underset{7}{\mid +}$ $\quad x<-1$ or $x>7$

Since D: $x^2-6x-6>0$ using quadratic for $x \approx -.87$ and 6.87

$\underset{-.87}{+ \mid} \; - \; \underset{6.87}{\mid +}$ so both intervals are in the domain

72. $\log_2 x = \log_x 3 \Rightarrow \dfrac{\log x}{\log 2} = \dfrac{\log 3}{\log x} \Rightarrow (\log x)^2 = (\log 3)(\log 2)$

$\log x = \pm\sqrt{(\log 3)(\log 2)} \approx 2.393, .418$

80. $3\ln x = \alpha + 3\ln\beta \Rightarrow \ln x^3 \; \ln\beta^3 = \alpha \Rightarrow \ln\dfrac{x^3}{\beta^3} = \alpha$

$\Rightarrow \dfrac{x^3}{\beta^3} = e^{\alpha} \Rightarrow x^3 = \beta^3 e^{\alpha} \Rightarrow x = \sqrt[3]{\beta^3 e^{\alpha}} = \beta e^{\alpha/3}$

81. $y = Ae^{kx} \Rightarrow \dfrac{y}{A} = e^{kx} \Rightarrow \ln\left(\dfrac{y}{A}\right) = kx \Rightarrow x = \dfrac{1}{k}\ln\left(\dfrac{y}{A}\right)$

88. $4^x = 3^{2x+1} \Rightarrow x\ln 4 = (2x+1)\ln 3 \Rightarrow x\ln 4 = 2x\ln 3 + \ln 3$

$\Rightarrow x\ln 4 - 2x\ln 3 = \ln 3 \Rightarrow x(\ln 4 - 2\ln 3) = \ln 3 \Rightarrow$

$x = \dfrac{\ln 3}{\ln 4 - 2\ln 3} \approx -1.355$

96. $f(x) = \ln\left(x + \sqrt{(x^2+1)}\right) \Rightarrow y = \ln\left(x + \sqrt{x^2+1}\right)$

$\Rightarrow e^y = x + \sqrt{x^2+1} \Rightarrow (e^y - x)^2 = \left(\sqrt{x^2+1}\right)^2 \Rightarrow$

$e^{2y} - 2xe^y + x^2 = x^2+1 \Rightarrow e^{2y} - 2xe^y = 1 \Rightarrow$

$e^{2y} - 1 = 2xe^y \Rightarrow x = \dfrac{e^{2y}-1}{2e^y} \Rightarrow f^{-1}(x) = \dfrac{e^{2x}-1}{2e^x}$